RING OF COMBINATORIAL POLYTOPES AND FORMAL GROUPS

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A combinatorial polytope is a class of combinatorially equivalent convex polytopes. We describe the results of a new approach based on a differential ring of combinatorial polytopes and show that *the ring of simple polytopes is a remarkable differential subring.*

The results obtained are applied to the description of generating series with specially chosen coefficients of the f-, h- and γ-polynomials of the following remarkable series of simple polytopes: simplices $\Delta = \{\Delta^n\}$, cubes $I = \{I^n\}$, associahedra $As = \{As^n = K_{n+2}\}$ (Stasheff polytopes), cyclohedra $Cy = \{Cy^n\}$ (Bott–Taubes polytopes), permutohedra $Pe = \{Pe^n\}$ and stellohedra $St = \{St^n\}$.

We obtain differential equations that define these series and find their solutions. Some of this results are based on results of A. Fenn.

Corollaries:

- the generating series of h-polynomials of the families Δ, I, Pe, and St are related by $I_h Pe_h = \Delta_h St_h$;
- the generating series Δ_h and Pe_h coincide with the exponential of formal group laws corresponding to the fundamental two-parameter Hirzebruch genera, which give the classical Todd genus for $t = 0$.