Non-Linear Aeroelastic Modelling and Prediction

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Outline

• Overview of Aeroelasticity
• Outline of several aeroelastic phenomena
• Flutter
  – Prediction of stability bounds
• Effect of non-linearities
  – Structural
  – Aerodynamic
  – Control system
• Limit Cycle Oscillations
  – Determination of LCO characteristics
• Future Directions
104 Years Ago in the USA

- Wright brothers were perfecting their “Flyer” at Kitty Hawk.

- Samuel Langley, backed by the Smithsonian Institute, attempted to fly his “Aerodrome” off a houseboat on the Potomac River.
Langley’s Tests

- Structural failure
Wright Brothers

- Success
- Wing warping for roll control
Langley - First Known Aeroelastic Failure

- Wings were not stiff enough
- "Divergence" – torsional loads overcome structural restoring forces
- "Aerodrome" rebuilt some years later by Curtis with stiffer wings – it flew
- Interaction of flexible structure and aerodynamic forces need to be considered
- Science of Aeroelasticity
Collar’s Aeroelastic Triangle

- Aerodynamic Forces
- Elastic Forces
- Inertia Forces
- Vibration

Static Aeroelasticity

Stability and Control
Aeroelastic Phenomena

- Mostly undesirable
- Often catastrophic
  - Flutter / Divergence
- Response
  - Gusts / Manoeuvres / Control surface inputs
  - Buffet
- Linear and non-linear response
- Key criteria for aircraft design and certification
  - Many (1000s) of cases need to be considered
- Still unable to accurately predict some types of behaviour
Aeroelastic Equations

- 2nd order differential equation cf. $M\ddot{x} + C\dot{x} + Kx = 0$
- Stiffness and damping change with speed and height
- Matrices of order 80 x 80
- Right hand side for gusts / control inputs
Flutter

- Violent unstable vibration often resulting in structural failure
- Two modes interact with each other
Aeroelasticity at its Worst
Prediction of Aeroelastic Stability

- Aeroelastic equations
  \[ A\phi + (\rho VB + D)\phi + (\rho V^2C + E)q = 0 \]

- First order form
  \[
  \begin{bmatrix}
  \phi \\
  \phi
  \end{bmatrix}
  - \begin{bmatrix}
  0 & I \\
  -A^{-1}(\rho V^2C + E) & -A^{-1}(\rho VB + D)
  \end{bmatrix}
  \begin{bmatrix}
  q \\
  \phi
  \end{bmatrix} = 0
  \]

- Eigenvalue problem
  \[ x - Qx = 0 \quad x = x_0 e^{\lambda t} \quad \Rightarrow \quad (Q - I\lambda) x_0 = 0 \]

- Eigenvalues of Q
  \[ \lambda_j = -\zeta_j \omega_j \pm i\omega_j \sqrt{1 - \zeta_j^2} \quad j = 1, 2, K, N \]
Unsteady Aerodynamics

- Steady lift proportional to angle of incidence
- Consider sudden change in incidence
Effect of Harmonic Motion

- Oscillatory motion of aerofoil
- Lift depends upon the reduced frequency
  \[ k = \frac{\omega b}{V} = \frac{\omega c}{2V} \]
- B and C are reduced frequency dependent
Frequency Matching

- Aeroelastic equations
  \[ A\ddot{\phi} + (\rho VB + D)\dot{\phi} + (\rho V^2 C + E)\phi = 0 \]
- If A, B, C, D, E are known
  -Typically using “panel methods”
  -Find \( \omega \) and \( \zeta \) from eigen problem
- But, need to know \( \omega \) and \( \zeta \)
  -To find B and C
- “Chicken and Egg” situation
  -Frequency matching problem
  -Consider individual harmonics
PK Method

- At each speed and frequency
  - Guess frequency
  - Calculate B and C
  - Solve eigenproblem
  - Repeat process with new frequency
- Exact at flutter condition
- Sub-critical behaviour not exact
Control Surface Flutter

- Interaction of control surface and wing
- F-117 Stealth Fighter
  - Freeplay of control surfaces
Effect of Non-Linearities

- Non-linear phenomena change aeroelastic behaviour
  - Structural
    - Cubic stiffening - joints
    - Freeplay – control surfaces
  - Aerodynamic
    - Transonic behaviour
    - Moving shocks
    - Stall flutter
  - Control
    - Control surface rate and deflection limits
    - Control circuit time delays
Effect of Transonic Flow on Flutter Speed

Need high fidelity aerodynamics to model accurately
Limit Cycle Oscillations

- Non-linearities cause a bounded flutter to occur
- Not disastrous
  - Fatigue problem
  - Other problems
    - weapon aiming
    - pilot control
Typical Parameter Space Behaviour
LCO Amplitude Behaviour

LCO on set for freeplay

Freeplay

Cubic Stiffness

Linear Flutter Speed

Velocity

LCO Amplitude
Limit Cycle Oscillations

- Interaction of control surface and wing
- Example here is limited amplitude
  - Limit Cycle Oscillation (LCO)
Stall Flutter

- If angle of incidence gets too high
  - Flow separates
  - Lift is lost
  - Incidence reduces
  - Flow reattaches
  - Incidence increases

- LCO results
- Occurs at wing tips
Military Aircraft with Stores

- Cannot predict LCO using current methods
- Problems if unpredicted vibration occurs in flight test
- Lots of (expensive) testing needed
Prediction of Stability Boundaries and Characteristics

- Possible to compute coupled FE/CFD models but very expensive in transonic region
- Many design cases need to be considered
- Aim to use non-linear dynamics methods to determine regions of interest
- Direct interesting areas where the FE/CFD analysis should be used
- Interested in
  - Stability boundaries
  - Amplitudes
  - Frequencies
Non-Linear Aeroelastic Prediction

- Continuous Non-linearities (Normal Form)
  - structural and aerodynamic non-linearities
  - determination of LCO frequencies and amplitudes

- Discontinuous Non-linearities (Normal Form / Harmonic Balance methods / Cell Mapping / numerical continuation)
  - structural and control system non-linearities
Aeroelastic Equations of Motion

\[ A \ddot{\Phi} + (\rho VB + D) \dot{\Phi} + (\rho V^2 C + E) q + F(q, \Phi) = 0 \]

\[
\begin{bmatrix}
\Phi \\
\Phi \\
q
\end{bmatrix} - \begin{bmatrix}
0 \\
-I \\
-A^{-1}(\rho V^2 C + E) & -A^{-1}(\rho VB + D)
\end{bmatrix} \begin{bmatrix}
q \\
q
\end{bmatrix} + \begin{bmatrix}
0 \\
F
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

- **F** – nonlinearity
- **Several methods used**
  - Normal Form
  - Higher Order Harmonic Balance
  - Numerical Continuation
Normal Form Theory

- Technique to obtain the local post-critical behaviour on a 1DOF undergoing Hopf Bifurcation
- Requires Centre manifold theory to simplify system from MDOF to 1DOF
- Applied to structural discontinuous non-linearities
  - Curve-fit the non-linearity
  - Define the equation of motion
  - Define the linear flutter condition
  - Reduce the model (Centre Manifold)
  - Apply Normal Form Theory
  - Determine amplitude and frequency of LCO
NFT Solution

- 18 DOF aeroelastic model
- Bi-linear non-linearity with various ratio of inner / outer stiffness
Higher Order Harmonic Balance

- Various types of bifurcation can be treated using Harmonic Balance methods
  - Hopf (sub-critical and super-critical)
  - Period Doubling
  - Folds
- Harmonic Balance
  - Approximates LCO with single sinusoidal component
  - Implementation
    - Describing function
    - Equivalent linearisation
  - Accuracy reduced if significant higher order components exists
- Higher Order Harmonic Balance
  - Similar to HB but higher order terms included
  - Approximates LCO with multiple sinusoids
BAH Aeroelastic Model

- Bisplinghoff, Ashley and Halfman (BAH) wing
- 12 Finite Element nodes, 72 degrees of freedom
- 9 modes
- Unsteady aerodynamics with 4 aerodynamic lags
- A total of 54 states
- Piecewise linear nonlinearity in control surface rotation degree of freedom
Bifurcation plots from HOHB

- Bifurcation plots from HB 5 and HB 17 for lower LCO branch
- 17th order HB results are more accurate but slower
Numerical Continuation

- Numerical Continuation is a method designed to solve nonlinear algebraic equations that depend on one or more parameters.
- Having achieved a solution, can then change the parameters slightly and track the new solution.

![Diagram showing stable and unstable equilibrium solutions](image-url)
NC applied to full aircraft

- 9 structural modes
- 4 aerodynamic lag roots
- DOF-2-DOF cubic / freeplay non-linearity applied to control surface
Frequency-damping plot

- M, C, K, AIC extracted from Nastran
- Roger approximation applied to transform AIC matrices to time domain
Cubic non-linearity
Conclusions

- A number of different approaches have been described for modeling and prediction of linear and non-linear aeroelastic behaviour
- Prediction of non-linear aeroelastic phenomena presents a number of challenges for analysis, testing and FE/CFD modelling
- Further work involves the extension to larger order models and transonic aerodynamics