The Appearance of $k-$th order polynomial eigenvalue problems

when damping is modeled by fractional derivatives

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Outline

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Definition

Fractional Differential Equations

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}} + \boldsymbol{C}\,\mathrm{D}^{\alpha}\,\boldsymbol{u} + \boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{0}, \quad \boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K} \in \mathbb{R}^{n\times n}. \tag{1}$$

Riemann-Liouville definition

$$\mathrm{D}^{\alpha}\boldsymbol{u}(t) = \frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{\boldsymbol{u}(t-\tau)}{\tau^{\alpha}}\mathrm{d}\tau, \quad 0 < \alpha < 1 \tag{2}$$

$$\alpha = \frac{p}{q}, \quad p, q \in \mathbb{N}_0^+ \tag{3}$$

Augmented State-Space Representation

Composition Rule [7]

$$\boldsymbol{A}\,\boldsymbol{z} = \boldsymbol{B}\,\mathrm{D}^{\alpha}\,\boldsymbol{z}, \quad \boldsymbol{A}\,,\boldsymbol{B} \in \mathbb{R}^{2\,q\,n\times 2\,q\,n}, \quad \alpha = \frac{1}{2} \tag{4}$$

Suarez and Shokooh et al. [9, 8, 1, 5]

$$\begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{I}_n & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{I}_n & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{I}_n & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & -\widetilde{\boldsymbol{K}} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \boldsymbol{z}_3 \\ \boldsymbol{z}_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{I}_n \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{I}_n & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{I}_n & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{I}_n & \boldsymbol{O} & \boldsymbol{O} & \widetilde{\boldsymbol{C}} \end{bmatrix} \mathrm{D}^{\alpha} \begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \boldsymbol{z}_3 \\ \boldsymbol{z}_4 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & -\boldsymbol{K} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \boldsymbol{z}_3 \\ \boldsymbol{z}_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C} \end{bmatrix} \mathrm{D}^{\alpha} \begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \boldsymbol{z}_3 \\ \boldsymbol{z}_4 \end{bmatrix} \tag{6}$$

$$\boldsymbol{z}_4 = \boldsymbol{u}, \quad \mathrm{D}^{\alpha}\boldsymbol{z}_1 = \ddot{\boldsymbol{u}}$$

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Augmented State-Space Representation

$$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{B}\,\mathrm{D}^{\alpha}\boldsymbol{z}, \quad \boldsymbol{A},\, \boldsymbol{B} \in \mathbb{R}^{2qn\times 2qn}, \quad \alpha = \frac{2}{3} \tag{7}$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C} \\ \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C} & \boldsymbol{O} \end{bmatrix}, \tag{8}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{M} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & -\boldsymbol{K} \end{bmatrix} \tag{9}$$

Bottleneck and a Loophole

- The system of equations tends to become very large when the denominator q of the order $\alpha = \frac{p}{q}$ of the derivative is very large.
- Applying the Fourier transform to the system of equations (1), one obtains

$$\left[\boldsymbol{M}\,(\mathrm{i}\,\omega)^{2} + \boldsymbol{C}\,(\mathrm{i}\,\omega)^{\alpha} + \boldsymbol{K}\right]\boldsymbol{x} = \boldsymbol{0}. \tag{10}$$

- It is convenient to denote the eigenvalues by $\lambda_i = (\mathrm{i}\,\omega_i)^{\frac{1}{q}}$

$$\left[\boldsymbol{M}\,\lambda^{2\,q} + \boldsymbol{C}\,\lambda^{p} + \boldsymbol{K}\right]\boldsymbol{x} = \boldsymbol{0}. \tag{11}$$

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Extension

Fenander (1996) [3]

$$l\, \boldsymbol{M}\, \mathrm{D}^{2+\alpha}\, \boldsymbol{u} + \boldsymbol{M}\, \ddot{\boldsymbol{u}} + \boldsymbol{C}\, \mathrm{D}^{\alpha}\, \boldsymbol{u} + \boldsymbol{K}\, \boldsymbol{u} = \boldsymbol{f} + l\, \mathrm{D}^{\alpha}\, \boldsymbol{f}. \tag{12}$$

Rational eigenvalue problem Voss (2006) [6]

$$\left(\boldsymbol{M}\lambda^{2q} + \frac{\boldsymbol{C}\,\lambda^{p} + \boldsymbol{K}}{1 + l\,\lambda^{p}} \right) \boldsymbol{x} = \boldsymbol{0}. \tag{13}$$

Polynomial eigenvalue problem

$$\left(l\,\boldsymbol{M}\lambda^{2q+p} + \lambda^{2q}\,\boldsymbol{M} + \lambda^{p}\,\boldsymbol{C} + \boldsymbol{K} \right) \boldsymbol{x} = \boldsymbol{0}. \tag{14}$$

Chu's Homotopy

Polynomial eigenvalue problem

$$\boldsymbol{P}(\lambda)\,\boldsymbol{x} = \boldsymbol{0} \tag{15}$$

$$\boldsymbol{P}(\lambda) = \boldsymbol{A}_k\,\lambda^k + \boldsymbol{A}_{k-1}\,\lambda^{k-1} + \cdots + \boldsymbol{A}_1\,\lambda + \boldsymbol{A}_0 \tag{16}$$

Nonlinear system $\boldsymbol{h}(\boldsymbol{x}, \lambda, t) = \boldsymbol{0}$ Chu (1988) [2]

$$\boldsymbol{h}(\boldsymbol{x}, \lambda, t) = \begin{bmatrix} \boldsymbol{R}(\lambda, t, \boldsymbol{D})\,\boldsymbol{x} \\ \frac{1}{2}(\boldsymbol{x}^{\mathrm{H}}\,\boldsymbol{x} - 1) \end{bmatrix} \tag{17}$$

$$\begin{aligned} \boldsymbol{R}(\lambda, t, \boldsymbol{D}, c) &= (1-t)\,\boldsymbol{Q}(\lambda) + t\,\boldsymbol{P}(\lambda), \\ \boldsymbol{Q}(\lambda) &= c\,\boldsymbol{I}_n\,\lambda^k - \boldsymbol{D}, \\ \boldsymbol{D} &= \mathrm{diag}(d_i), \quad c \in \mathbb{C}, \quad d_i \in \mathbb{C}. \end{aligned} \tag{18}$$

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Chu's Homotopy continued

$$\begin{bmatrix} \boldsymbol{R}(\lambda, t, \boldsymbol{D}, c) & \boldsymbol{R}_\lambda(\lambda, t, c)\,\boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{Q}(\lambda) - \boldsymbol{P}(\lambda))\,\boldsymbol{x} \\ 0 \end{bmatrix} \tag{19}$$

$k\,n$ initial conditions $\boldsymbol{z} = \begin{bmatrix} \boldsymbol{x}_i \\ \lambda_{ij} \end{bmatrix}$ with

$$\begin{aligned} \boldsymbol{x}_i(t=0) &= \boldsymbol{e}_i, & (20) \\ \lambda_i(t=0) &= \lambda_{ij} \quad i = 1, \ldots, n, \quad j = 1, \ldots, k. & (21) \end{aligned}$$

The *initial* eigenvalues λ_{ij} for $t = 0$ follow from

$$\det \boldsymbol{Q}(\lambda) = \prod_{i=1}^{n} \left(c - \lambda^k\, d_i \right) = 0. \tag{22}$$

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Chu's Homotopy continued

$$\begin{bmatrix} \boldsymbol{R}(\lambda, t, \boldsymbol{D}, c) & \boldsymbol{R}_\lambda(\lambda, t, c)\,\boldsymbol{x} \\ \boldsymbol{x}^{\mathrm{H}} & 0 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \lambda \end{bmatrix} = - \begin{bmatrix} \boldsymbol{R}(\lambda, t, \boldsymbol{D}, c)\,\boldsymbol{x} \\ \frac{1}{2}\left(\boldsymbol{x}^{\mathrm{H}}\,\boldsymbol{x} - 1\right) \end{bmatrix}. \quad (23)$$

$$\begin{aligned} \boldsymbol{P}_\lambda &= k\,\lambda^{k-1}\boldsymbol{A}_k + (k-1)\,\lambda^{k-2}\,\boldsymbol{A}_{k-1} + \cdots + 2\,\lambda\,\boldsymbol{A}_2 + \boldsymbol{A}_1, \\ \boldsymbol{Q}_\lambda &= c\,k\,\lambda^{k-1}\,\boldsymbol{I}_n, \\ \boldsymbol{R}_\lambda(\lambda, t, c) &= (1-t)\,\boldsymbol{Q}_\lambda(\lambda) + t\,\boldsymbol{P}_\lambda(\lambda). \end{aligned} \quad (24)$$

- The coefficient matrix in Eq. (19,23) is of order $n+1$.
- The parameters c, $\boldsymbol{D}$ in Eq. (18) are random numbers.
- The homotopy curves correspond only to different initial values of the same ODE.
- Hence, all curves can be followed simultaneously.

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Chu's Homotopy continued

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Example I

Agrawal (2002) [1]

$$m\,\ddot{u} + c\,\mathrm{D}^{0.5}\,u + k\,u = 0 \tag{25}$$

$$\boldsymbol{A}\,\boldsymbol{X} = \boldsymbol{B}\,\boldsymbol{X}\,\boldsymbol{\Lambda}, \quad \boldsymbol{X}^{\mathrm{T}}\,\boldsymbol{B}\,\boldsymbol{X} = \boldsymbol{I}_4, \quad \boldsymbol{\Lambda} = \mathrm{diag}(\lambda_i), \tag{26}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k/m \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & c/m \end{bmatrix} \tag{27}$$

$$u(t) = \frac{1}{\sqrt{\pi\,t}} \sum_{j=1}^{4} \boldsymbol{e}_4^{\mathrm{T}}\,\boldsymbol{x}_j\,c_j + \sum_{j=1}^{4} \lambda_j\,\boldsymbol{e}_4^{\mathrm{T}}\,\boldsymbol{x}_j\,g_j(t)\,c_j, \tag{28}$$

$$\dot{u}(t) = \frac{1}{\sqrt{\pi\,t}} \sum_{j=1}^{4} \boldsymbol{e}_2^{\mathrm{T}}\,\boldsymbol{x}_j\,c_j + \sum_{j=1}^{4} \lambda_j\,\boldsymbol{e}_2^{\mathrm{T}}\,\boldsymbol{x}_j\,g_j(t)\,c_j. \tag{29}$$

Example I continued

Since $u(t)$ and $\dot{u}(t)$ remain bounded as $t \to 0$, we have constraints for c_j. Finally we obtain a closed form solution for the position and velocity in terms of eigenvalues and eigenvectors [9].

$$u(t) = \sum_{j=1}^{4} \lambda_j (\boldsymbol{e}_4^{\mathrm{T}} \boldsymbol{x}_j)^2 \, (\dot{u}_0 + \lambda_j^2 \, u_0) \, g_j(t), \tag{30}$$

$$\dot{u}(t) = \sum_{j=1}^{4} \lambda_j^3 (\boldsymbol{e}_4^{\mathrm{T}} \boldsymbol{x}_j)^2 \, (\dot{u}_0 + \lambda_j^2 \, u_0) \, g_j(t). \tag{31}$$

$$g_j(t) = \exp(\lambda_j^2 \, t) \left(1 + \mathrm{erf}(\lambda_j \sqrt{t})\right) \tag{32}$$

Example I continued

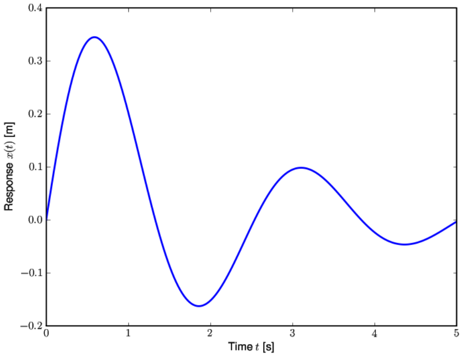

Figure: Free response of a fractionally damped oscillator, $u_0 = 0, \dot{u}_0 = 1.0\,\mathrm{m/s}$

Example II

Fenander (1996) [3]

$$\boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix},$$
$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}.$$

$$m_1 = 1\mathrm{kg},\ m_2 = 2\mathrm{kg}, \quad ,k_1 = k_3 = 1\mathrm{N/m},\ k_2 = 1.5\mathrm{N/m},$$
$$c_1 = c_2 = c_3 = 0.4\mathrm{Ns}^{2/3}/\mathrm{m},$$

- $\alpha = \frac{2}{3}$, $l = 0$ Eq. (11)
- $\alpha = \frac{1}{2}$, $l = 0.1\mathrm{s}^{1/2}$ Eq. (14).

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Numerical Results

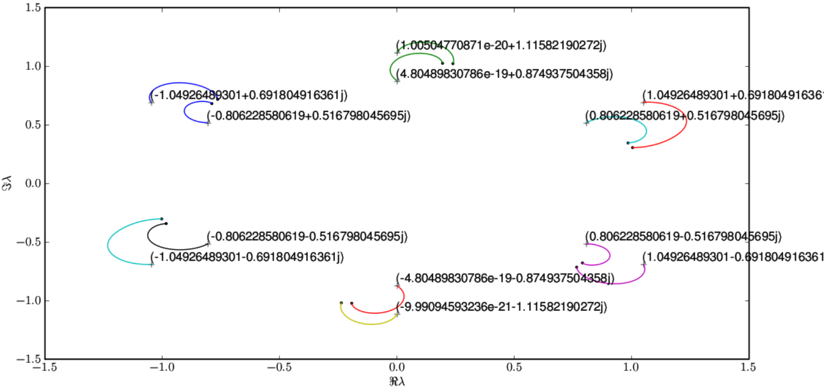

Figure: Homotopy curves for $\alpha = \frac{2}{3}, \quad l = 0$

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Conclusions and Outlook

- A direct approach based on a homotopy as suggested by Chu is applied to solve the polynomial eigenvalue problem.
- A predictor corrector method is used to solve the nonlinear problem (17).
- ZVODE[1] might be another option to solve the complex initial value problem (19).
- Direct time-domain solutions are very time-consuming due to the non-local character of fractional damping [10].
- All computation were carried out by SciPy [4].

[1] http://www.llnl.gov/CASC/software.html (see VODE)

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Thank you for your attention

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Software

http://numpy.scipy.org/

http://www.scipy.org/

http://matplotlib.sourceforge.net/