Particulate pipe flows

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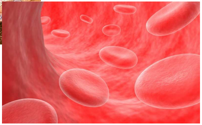
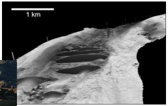
EFMC7 Manchester 2008

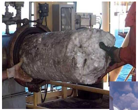
Particulate and granular flows







1. Neutrally-buoyant particles
   - Collective migration
   - Inertial migration
   - Transition to turbulence

2. Bed constituted of sediment particles
   - Incipient motion
   - Bed-load transport
   - Dune formation

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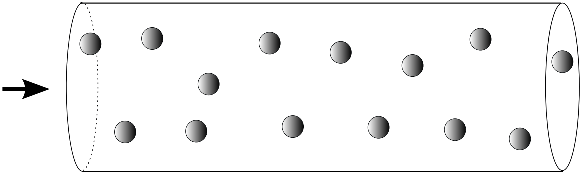
Neutrally-buoyant rigid particles in a pipe flow



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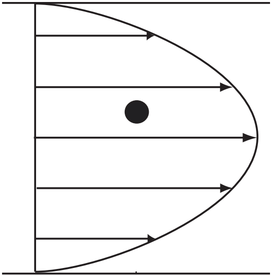
A single sphere in a Poiseuille flow at low Re



- Neutrally-buoyant sphere
- Migration?
- Reversibility
- Symmetry

  $\rightarrow$ No migration

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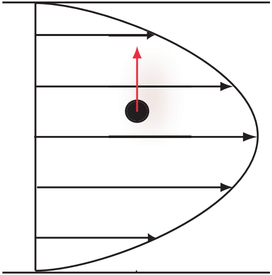

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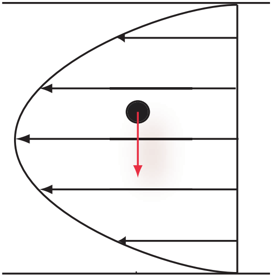

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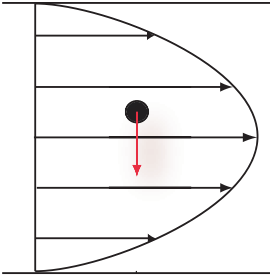

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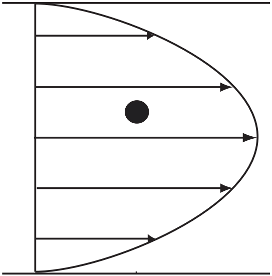
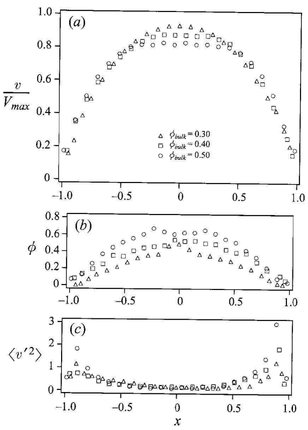
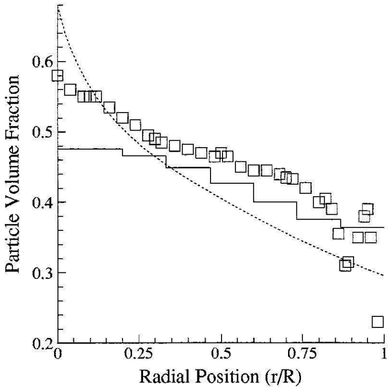


- Neutrally-buoyant sphere
- Migration?
- Reversibility
- Symmetry
  $\rightarrow$ No migration

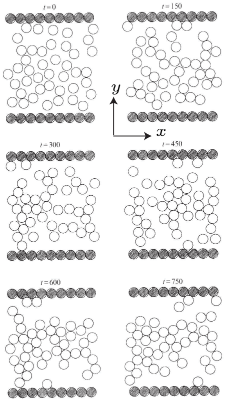
But collective migration at low Re!





Lyon & Leal 1998, Hampton *et al.* 1997, Butler & Bonnecaze 1999

Shear-induced migration



Nott & Brady 1994

- Diffusive flux model (Leighton & Acrivos 1987, Phillips *et al.* 1992)
- Suspension balance model (Nott & Brady 1994, Morris & Boulay 1999)
  - Steady fully developed flow in the x-direction with variation of properties in the y-direction
  - Particle y-momentum balance $\frac{\partial \Pi}{\partial y} = 0$
  - Viscous scaling $\Pi \sim \eta \dot{\gamma}(y) \bar{p}(\phi)$

Particle migration from regions of high shear to low shear

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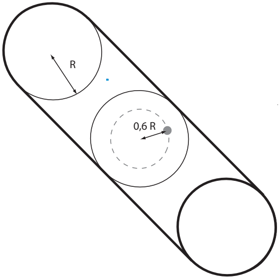
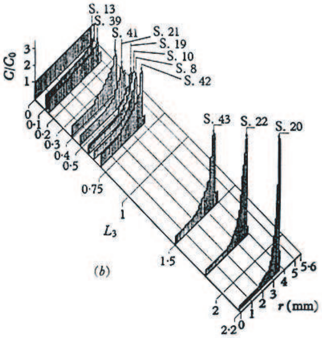
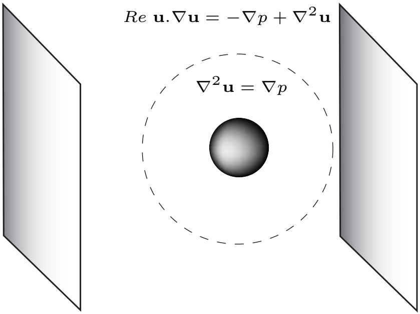
Tubular pinch effect





Segré & Silberberg 1962

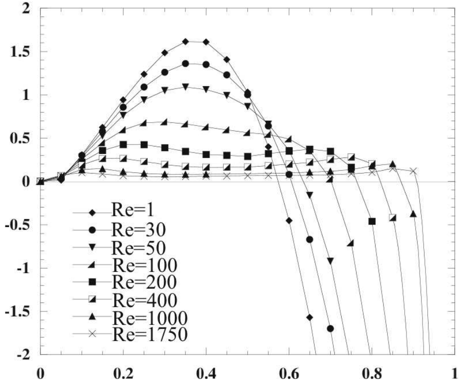
Matched asymptotic expansion theory

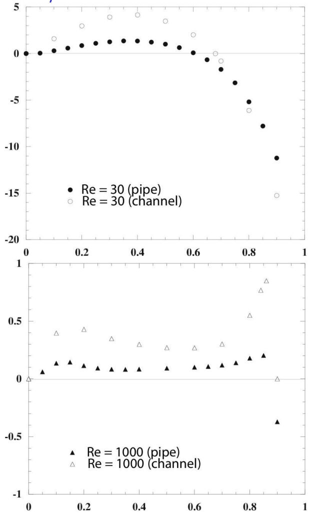


Inner viscous (Stokes) solution close to the sphere matched to outer inertial (Oseen) solution (small parameter $\epsilon = \sqrt{Re_p/2}$)

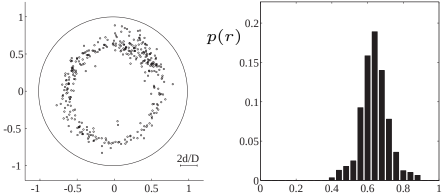
- Channel: Schonberg & Hinch 1989, Hogg 1994, Asmolov 1999
- Pipe: Matas, Morris & Guazzelli 2008

Normalised theoretical lift force versus r/R

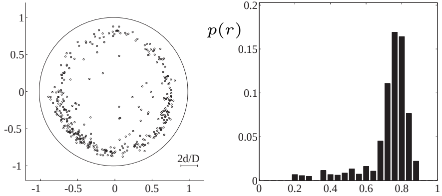




Segré-Silberberg annulus

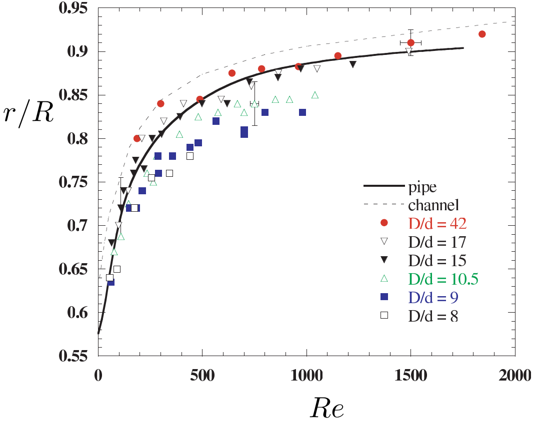


$Re = 70$



$Re = 350$

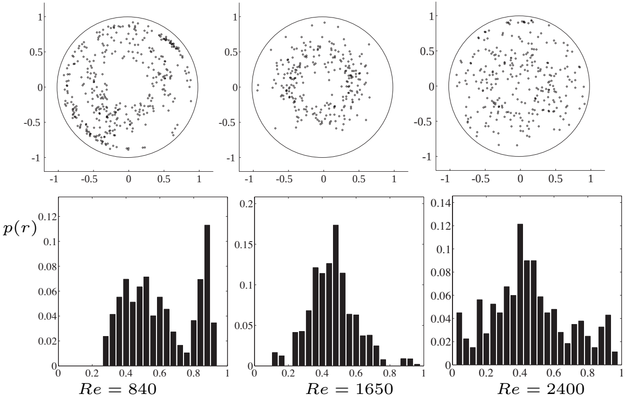
Experiment versus theory



For $Re_p = Re(\frac{d}{D})^2 \not< 1$: finite-size effects

Matas, Morris & Guazzelli 2004, 2008

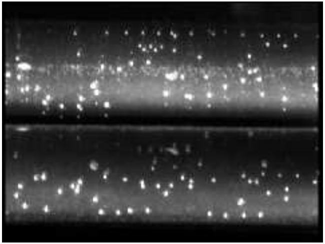
Inner and outer annulus



also seen in numerical simulations of Shao, Yu & Sun 2008

→ inner annulus most likely due to finite-size effects

Trains of particles located on the Segré-Silberberg annulus



Alignment attributable to hydrodynamic interactions associated with finite-inertia disturbance flows of particles in shear flow
Matas, Glezer, Guazzelli & Morris 2004
also seen in lattice-Boltzmann simulations of Chun & Ladd 2006

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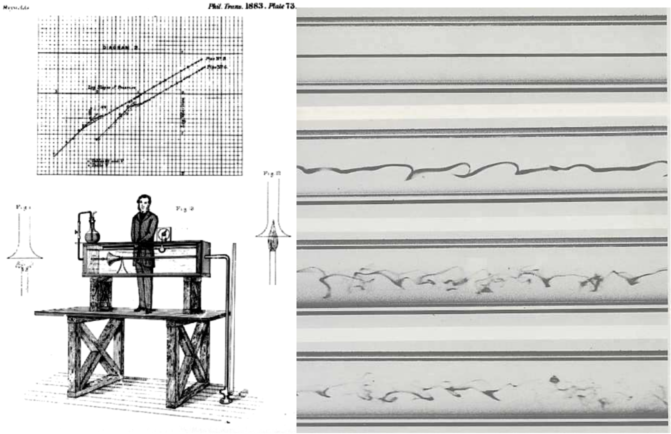
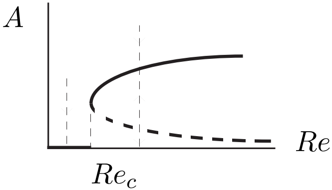
Transition to turbulence in pipe flow



Reynolds 1883

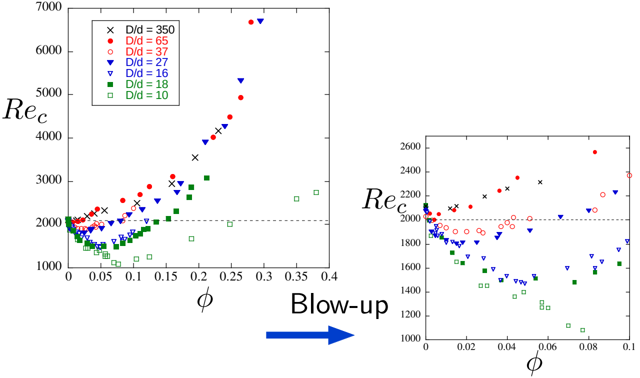
Subcritical transition



Flow linearly stable for all flow rates
Subcritical transition $\rightarrow$ a finite amplitude perturbation is needed to trigger the transition

Strong perturbation $\rightarrow$ intermittent regime (growth of turbulent *puff*) above $Re_{c0} \approx 2000$ for pure fluid

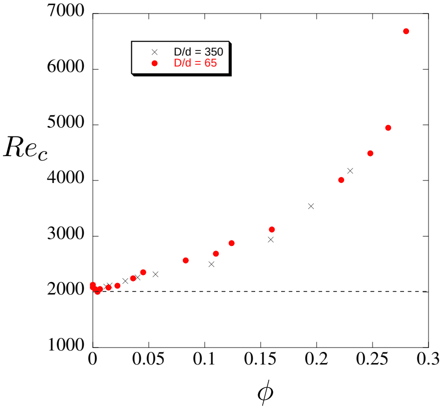
Influence of neutrally-buoyant spheres



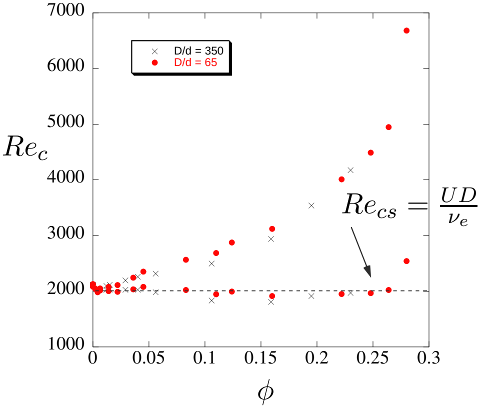
- $D/d \geq 65$ ($Re_p = Re\, d^2/D^2 \leq 1$) $\Rightarrow$ delayed transition
- $D/d \leq 65$ ($Re_p \geq 1$) $\Rightarrow$ lowered (then delayed) transition

Matas, Morris & Guazzelli 2003

Effective viscosity for $D/d \geq 65$

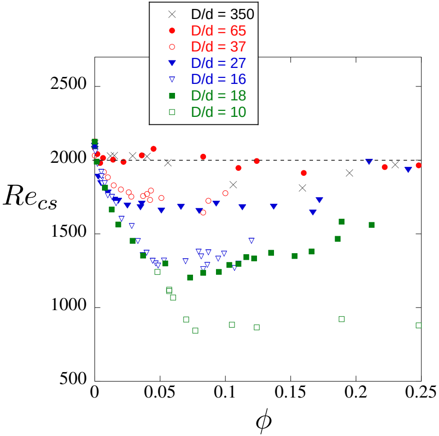


Effective viscosity for $D/d \geq 65$



Effective viscosity: $\nu_e = \nu_0(1 - \phi/\phi_m)^{-1.82}$ with $\phi_m = 0.68$
Krieger 1972

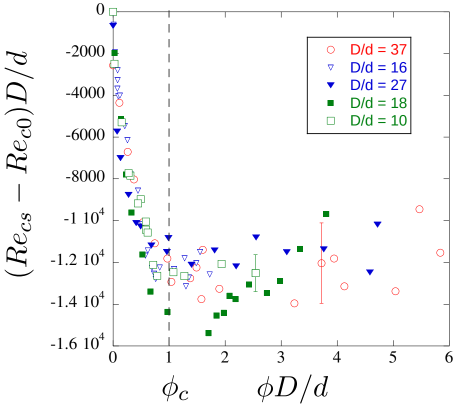
Critical Re using effective viscosity



- Linear decrease with ϕ at low ϕ
- Saturated minimum for larger ϕ

Not sufficient to obtain a collapse of the curves for $D/d \leqslant 65$!

Scaling the departure from Re_{c0}



- Linear decrease with $\phi D/d$
- Saturation for $\phi_c \approx d/D$

Large particles ($Re_p \not< 1$) can trigger the subcritical transition

But by which detailed mechanism?

Connection with travelling waves possibly related to transition?

Faisst & Eckhardt 2003, Wedin & Kerswell 2004, Hof *et al.* 2004

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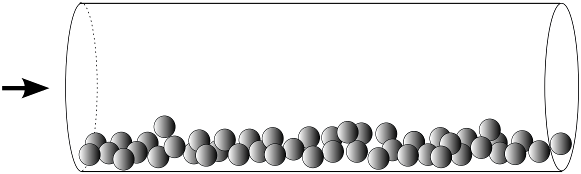
Bed constituted of sediment particles in a pipe flow



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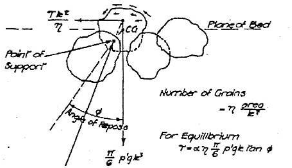
Incipient motion characterised by critical Shields number



White 1940, Vanoni 1966, ...

- Shields number: $\theta = \frac{\tau}{(\rho_p - \rho_f)gd}$
- Force balance on a grain:

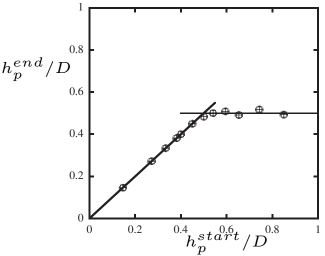
$$\begin{aligned} \theta^c &\propto \text{tangent of angle of repose} \\ &\propto \mu \\ &\approx 0.1 \end{aligned}$$

Experimental observations: $0.05 < \theta^c < 0.25$ in laminar flow

Large scatters due to:

- Bed packing conditions
- Multiple possible definition for the onset of grain motion
- Different definition of the shear stress

Threshold characterised through cessation of motion



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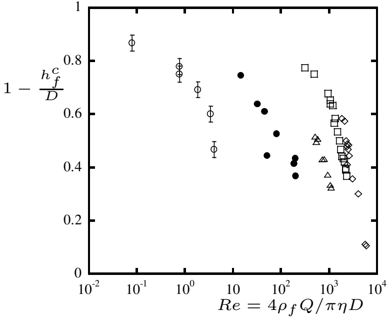
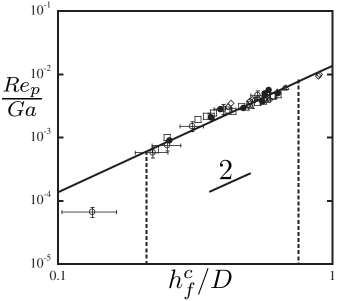


Onset for cessation of motion
→ constant critical shear stress

Not sufficient to obtain a collapse of the curves!

Critical Shields number for particle erosion



- Shields number: $\theta = \frac{\eta\dot{\gamma}}{(\rho_p - \rho_f)gd}$
- Shear rate:
  - $\dot{\gamma}_{2D} = 6\frac{Q_{2D}}{D^2}(\frac{D}{h_f})^2$
  - $\dot{\gamma}_{pipe} = \frac{Q_{pipe}}{D^3}f(\frac{D}{h_f}) = 6\beta\frac{Q_{pipe}}{D^3}(\frac{D}{h_f})^2$ with numerical $\beta = 1.85 \pm 0.02$
- Scaling: $\frac{Re}{Ga}(\frac{d}{D})^2 = \frac{Re_p}{Ga} = \frac{2}{3\pi\beta}\theta(\frac{h_f}{D})^2$ with:
  - $Ga = \frac{(\rho_p - \rho_f)\rho_f g d^3}{\eta^2}$
  - $Re_p = Re(\frac{d}{D})^2$

$\theta^c = 0.12 \pm 0.03$ in the range $1.5\ 10^{-5} \leqslant Re_p \leqslant 0.76$

Ouriemi, Aussillous, Medale, Peysson & Guazzelli 2007
in agreement with Charru *et al.* 2004 and Loiseleux *et al.* 2005

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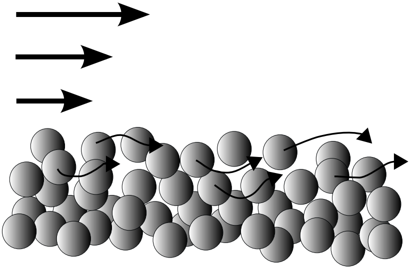
Bed-load transport

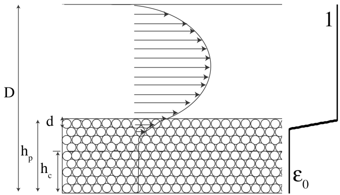


Einstein 1942, 1950, Bagnold 1956, Yalin 1963 ...

Viscous flow: Charru & Mouilleron-Arnould 2002, Charru, Mouilleron & Eiff 2004, Charru & Hinch 2006

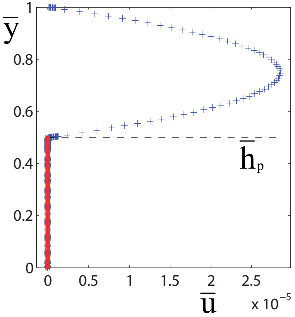
Two-phase model of bed-load transport

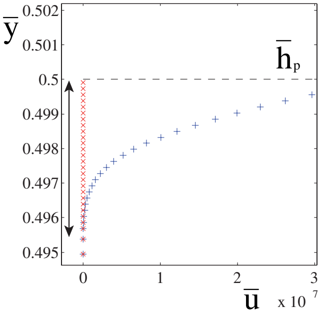
- Continuity equations for the fluid and particle phases
- Momentum equations for the fluid and particle phases
  - Particle-fluid interaction: Darcy + Buoyancy
  - Newtonian rheology for the fluid phase (Einstein viscosity)
  - Coulomb friction for the particle phase (friction coefficient μ)



- Brinkman equation for the fluid phase (Darcy term dominant)
- Mixture (fluid + particles) momentum equation (exchange between stresses of the fluid and solid phases)

Numerical velocity-profiles for $\theta = 0.05$

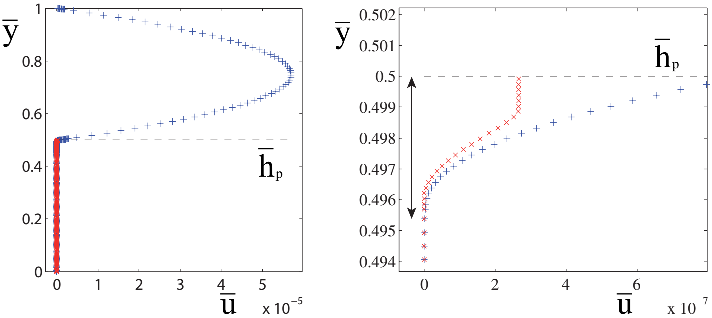




No motion of the solid phase

Ouriemi, Aussillous & Guazzelli 2008

Numerical velocity-profiles for $\theta = 0.1$

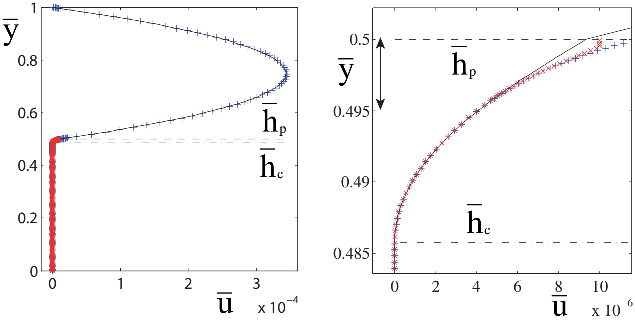


Motion of a thin layer of solid phase ($\lesssim$ one particle diameter)

Numerical $\theta^c \approx 0.06$ smaller than experimental $\theta^c = 0.12$

Continuum model only very qualitative just above incipient motion!

Numerical and analytical velocity-profiles for $\theta = 0.6$

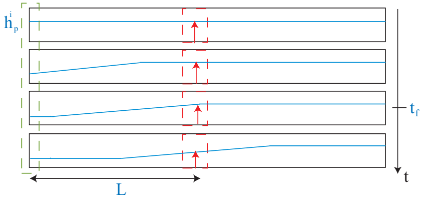


No slip between fluid and solid phases

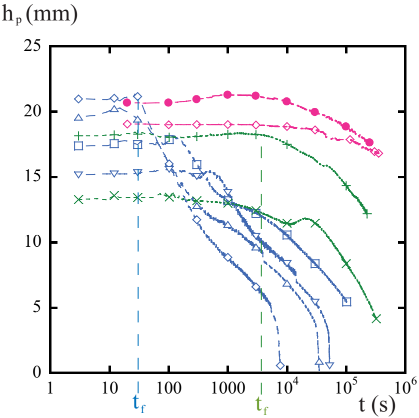
Bed-load flow-rate for $0.5 \lesssim \theta \lesssim 1.5$: $q_p / \frac{\Delta\rho g d^3}{\eta_e} = \phi_0 \frac{\theta^c}{24} \left(\frac{\theta}{\theta^c}\right)^3$

Bed profile evolution

No direct measurement of particle flux but evolution of the bed height



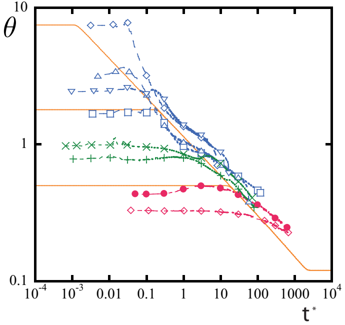
Test section not fed in with particles



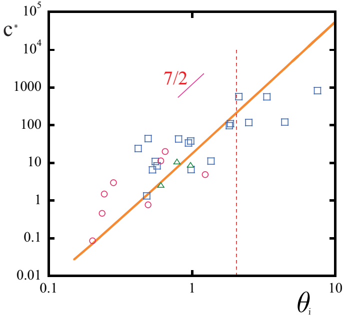
Comparison with particle flux prediction $q_p \propto \theta^3$

Mass conservation: $\phi_0 \frac{\partial h_p}{\partial t} + \frac{\partial q_p}{\partial x} = 0 \Rightarrow$ kinematic wave equation

$$\frac{\partial \theta}{\partial t^*} + c^*(\theta) \frac{\partial \theta}{\partial x^*} = 0$$



$$c^*(\theta) \propto \frac{\partial q_p}{\partial \theta} \propto \theta^{7/2}$$



$$\theta^c = 0.12, \; \eta_e = \eta(1 + 5\phi_0/2)$$

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Bed regimes and dune patterns

$\downarrow Re$

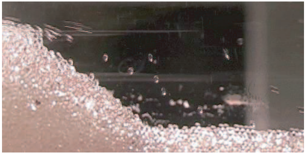
- No motion
- Flat bed in motion

$\Rightarrow$

- Small dunes

- Vortex dunes

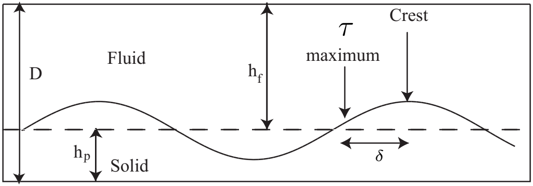
$\Rightarrow$



- Sinuous dunes

Ouriemi, Aussillous & Guazzelli 2008

Destabilising mechanism: fluid inertia

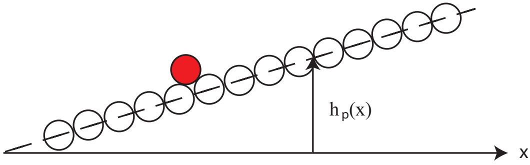


Fluid inertia → phase-lag between shear stress and bed waviness

The shear stress, the maxima of which are slightly shifted upstream of the crests, drags the particles from the troughs up to the crests

Kennedy 1963, Charru and Hinch 2000

Stabilising mechanism: gravity

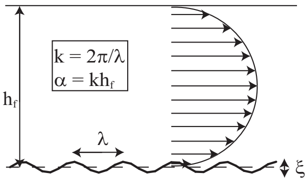


Gravity force favours particle downhill motion

Shift of the critical Shields number for incipient motion:
$\theta^c = \theta_0^c(1 + \frac{\partial h_p/\partial x}{\mu})$ with μ friction coefficient

Fredsoe 1974, Richards 1980, Charru & Hinch 2006

A simple linear stability analysis (Charru & Hinch 2000)



$$\xi = \xi_1 e^{i(kx-\omega t)}$$

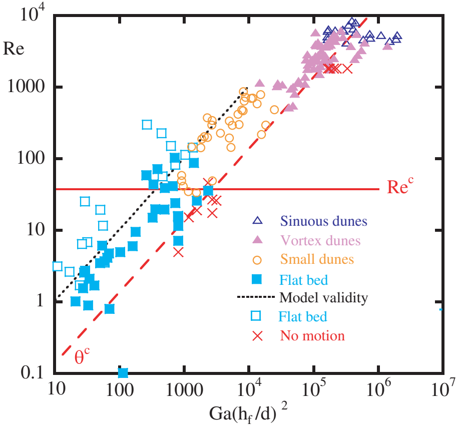
- Mass conservation: $\frac{\partial q_p}{\partial x} + \phi_0 \frac{\partial \xi}{\partial t} = 0$
- Particle flux: $q_p \propto \frac{\theta^3}{\theta^{c2}}$
- θ from $\dot{\gamma}$ calculated at the top of the fixed wavy bottom with basic ingredients of:
  - destabilising fluid inertia
  - stabilising gravity

Threshold for dune instability:

2D $\rightarrow$ $Re^c_{2D} = \frac{70}{3\mu}$

Pipe $\rightarrow$ $Re^c = \frac{280}{3\beta\pi\mu}$ with numerical $\beta = 1.85$

Phase diagram of the dune patterns



- Incipient motion:

$$Re \propto \theta^c Ga \left(\frac{h_f}{d}\right)^2$$

- Instability threshold:

$$Re^c \approx 37.5$$

with $\mu = 0.43$

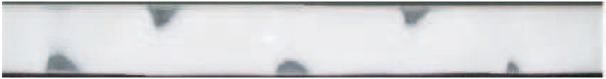
Nonlinear and turbulent . . .

- Vortex dunes

Top view

Side view

- Sinuous dunes



Top view

Side view

1. Neutrally-buoyant particles
   - Collective migration
   - Inertial migration
   - Transition to turbulence

2. Bed constituted of sediment particles
   - Incipient motion
   - Bed-load transport
   - Dune formation

Conclusions

Particulate multiphase flows offers problems of far great complexity than found in single-phase flows. This leads to many new and intriguing flow phenomena absent in the single phase flows.

- Collective and inertial migration
- Transition to turbulence in particulate pipe flow
- Particle erosion and bed-load transport
- Dune patterns

Collaborations and thanks

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