Optimi~~z~~sation of Uniflagellate and Biflagellate Locomotion

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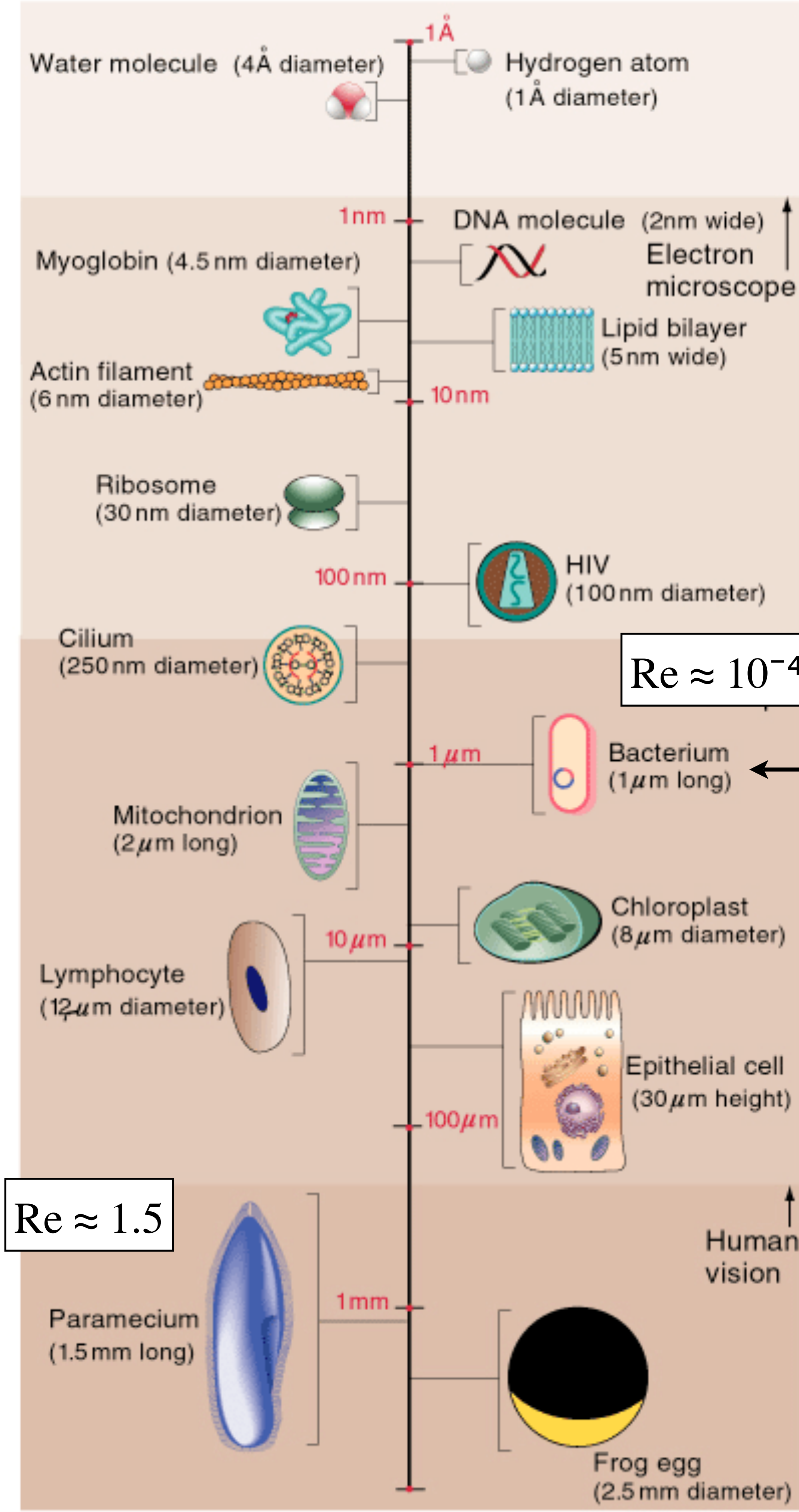
Low Reynolds Numbers

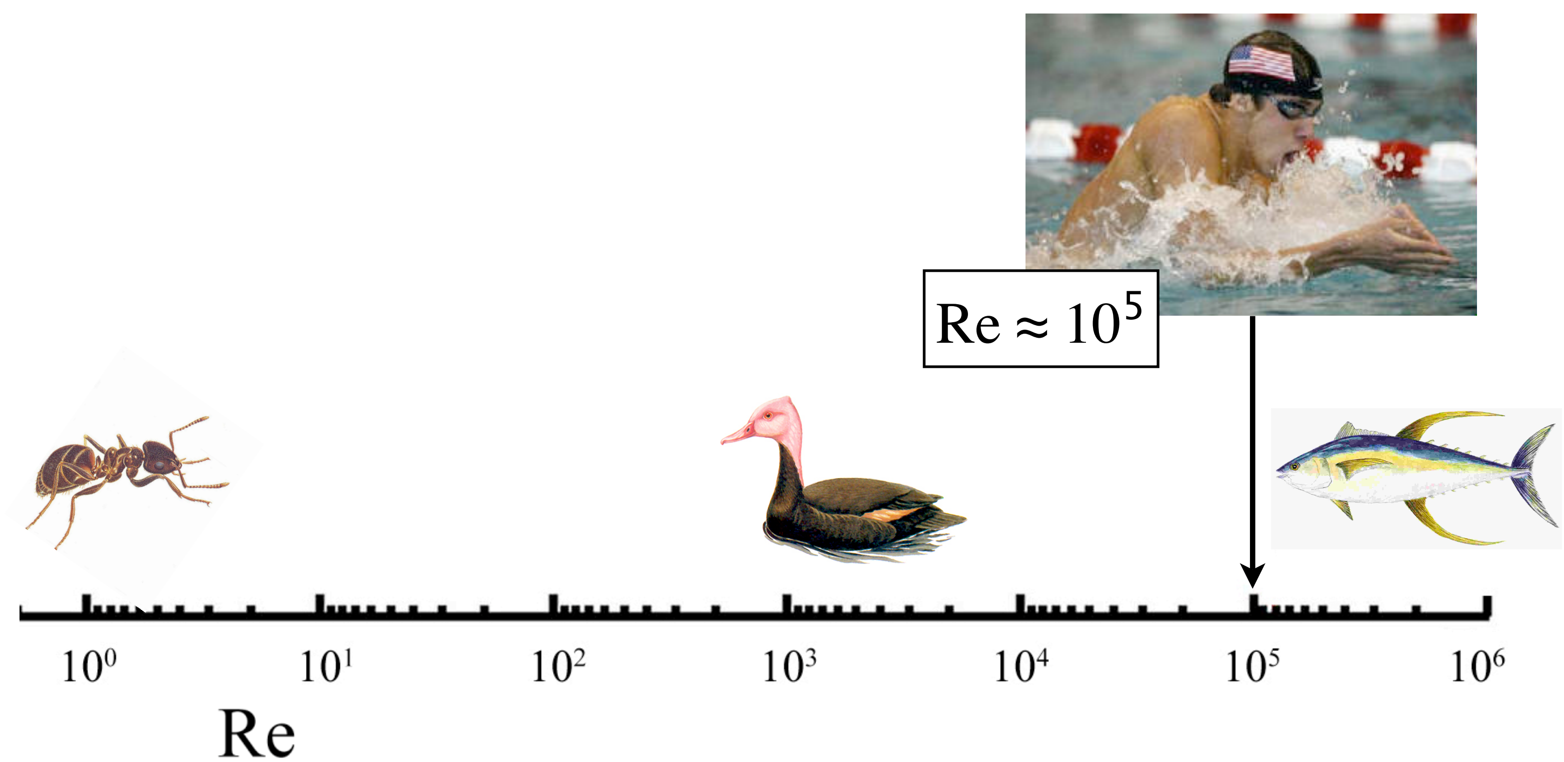
$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho U L}{\mu}$$



$\mathrm{Re} \approx 10^{-4}$

$\mathrm{Re} \approx 1.5$

"The smallest thing you can see with a microscope."



Taxonomy of Microorganisms

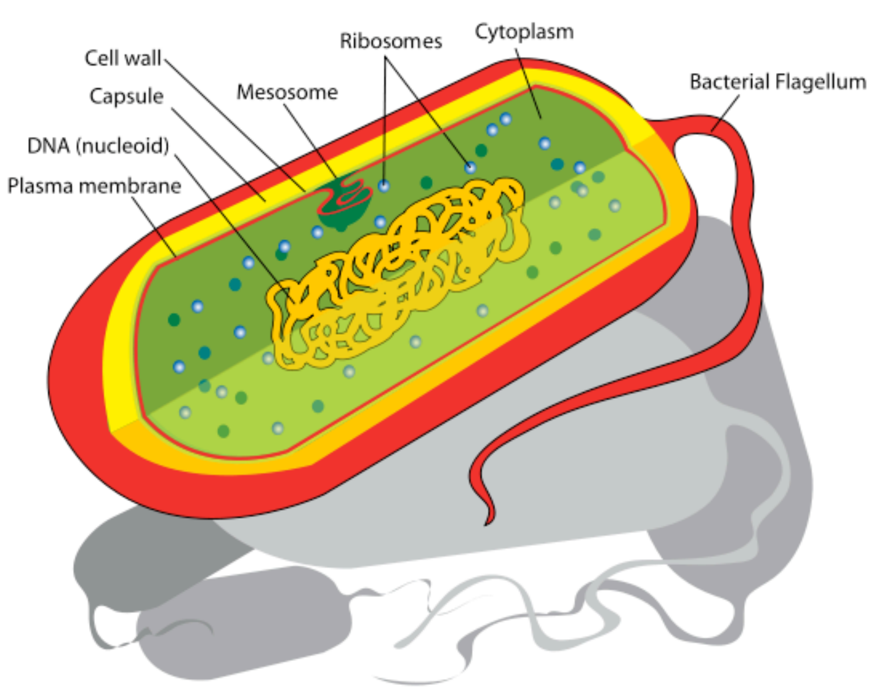
FLAGELLAR HYDRODYNAMICS*

The John von Neumann Lecture, 1975

JAMES LIGHTHILL†

SIAM Review

Vol. 18. No. 2. April 1976



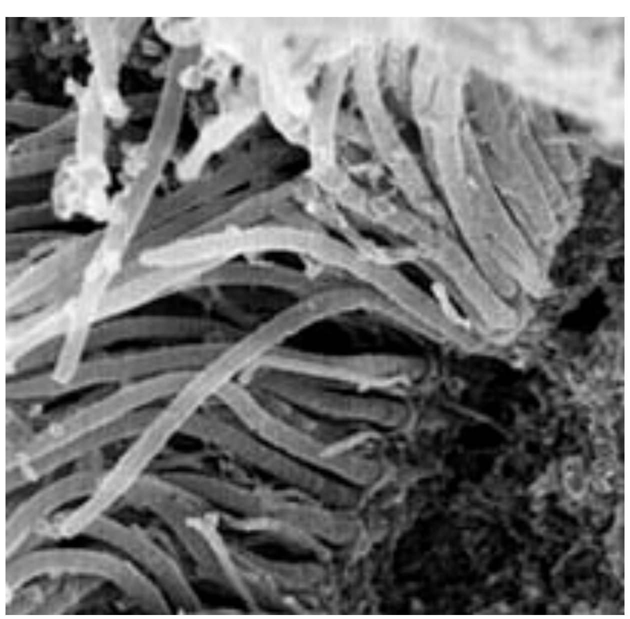
continuous gradient between plantlike and animal-like organisms

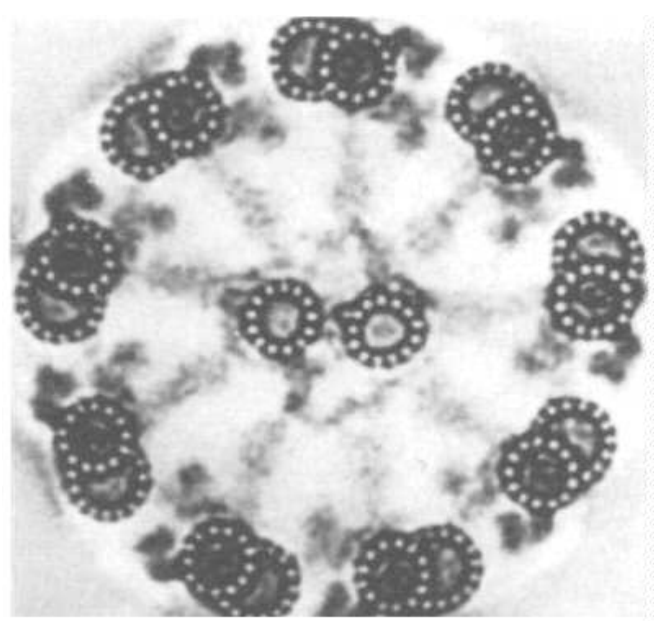
Sharp boundary between prokaryotic (no membrane-bound organelles) and eukaryotic (membrane-bound nuclei and other organelles)

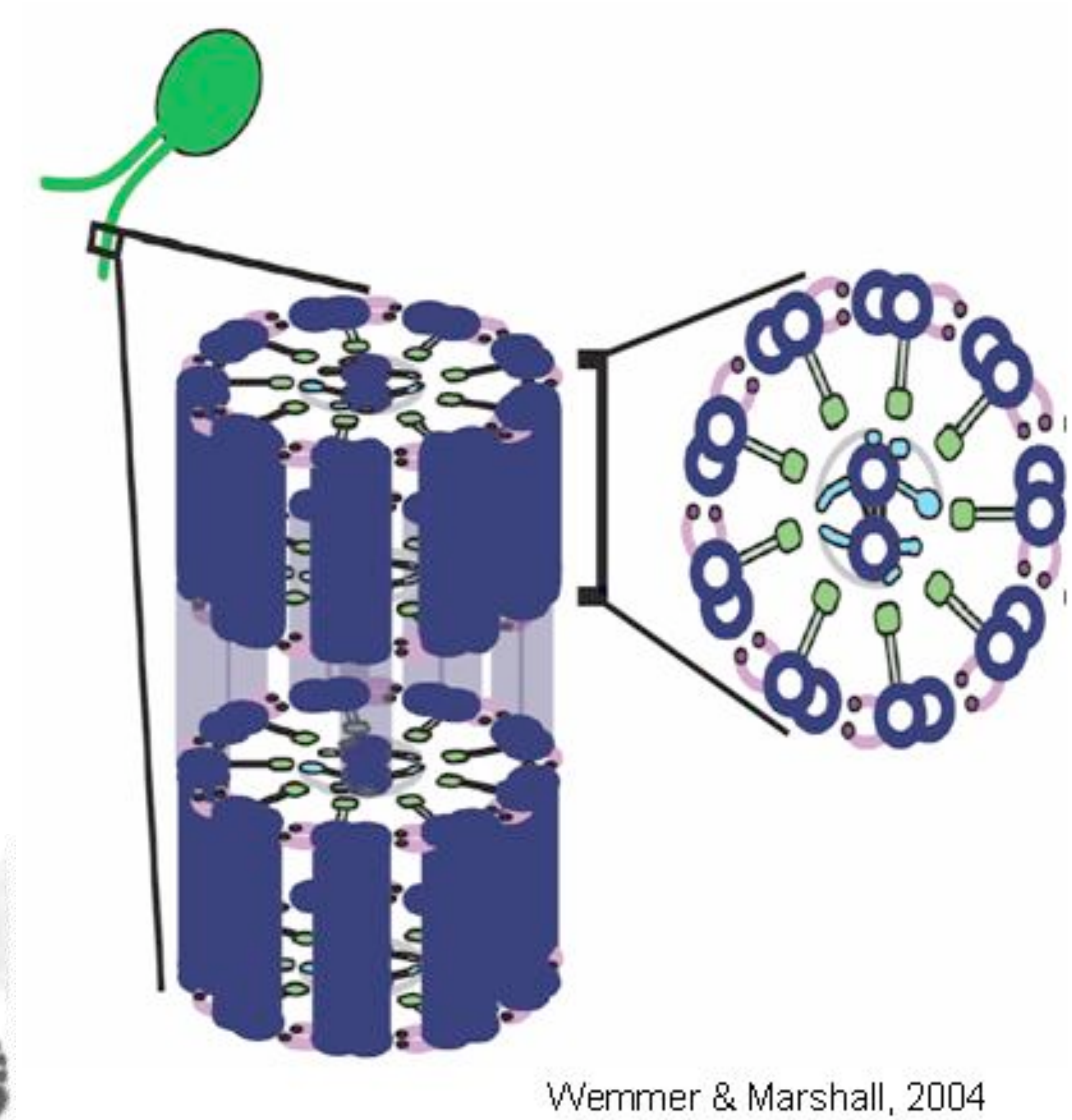
Uniflagellate vs biflagellate

Structure of Flagella and Cilia

- Eukaryotic cells (flagella and cilia)
  - 9+2 microtubule structure
  - Diameter of tail ≈ 250-400 nm ≈ constant across ALL species!
  - Organism can apply local bending moments along the tail ➞ can select shape as a function of time (control kinematics)

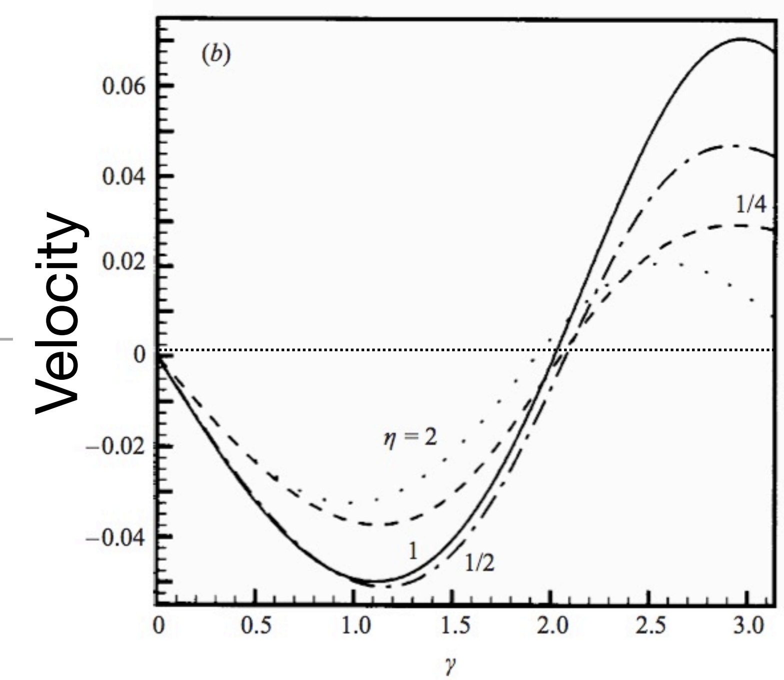






http://sun.menloschool.org/~cweaver/cells/e/cilia_flagella/
http://cellbio.utmb.edu/cellbio/cilia.htm

Simple Model System: 3-link Swimmer

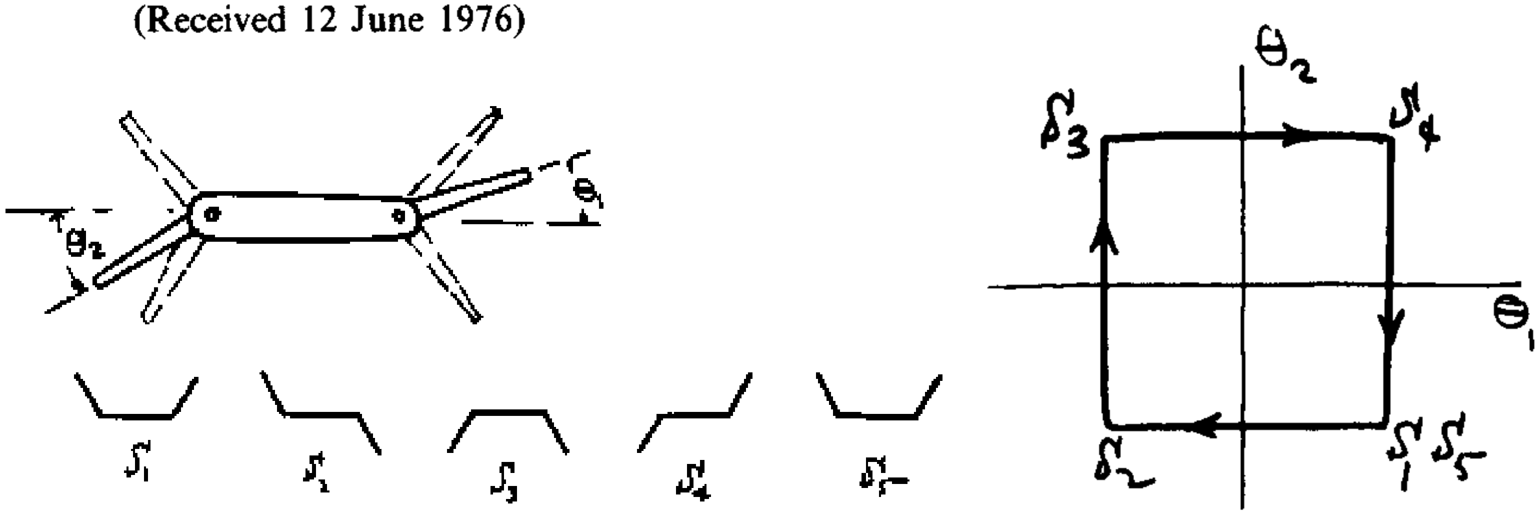


Life at low Reynolds number

E. M. Purcell

Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138

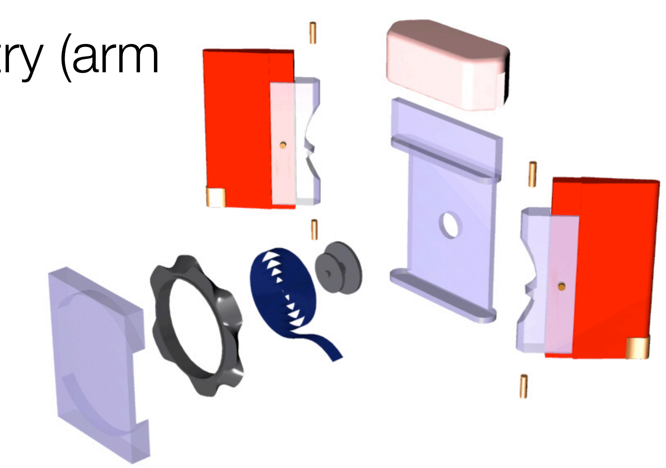
(Received 12 June 1976)



- **Purcell (1977):** proposed design
  - “In fact, I worked this one out just for fun and you can prove from symmetry that it goes along the direction shown in the figure. As an exercise for the student, what is it that distinguishes that direction?”

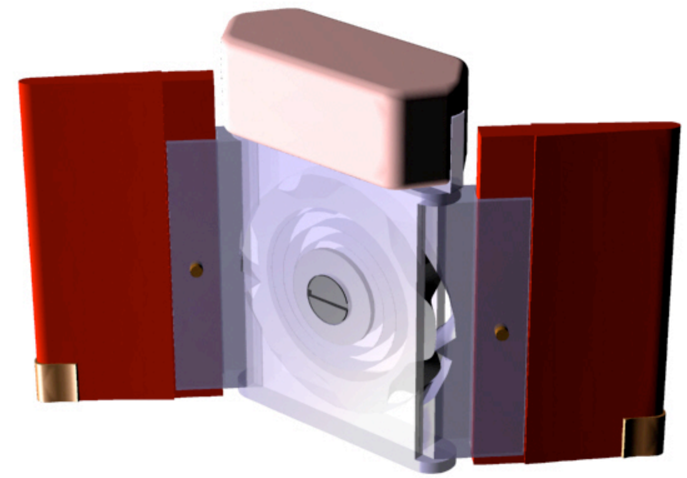
- **Becker, Koehler and Stone (2003):** optimized geometry (arm length/body length and stroke angle)

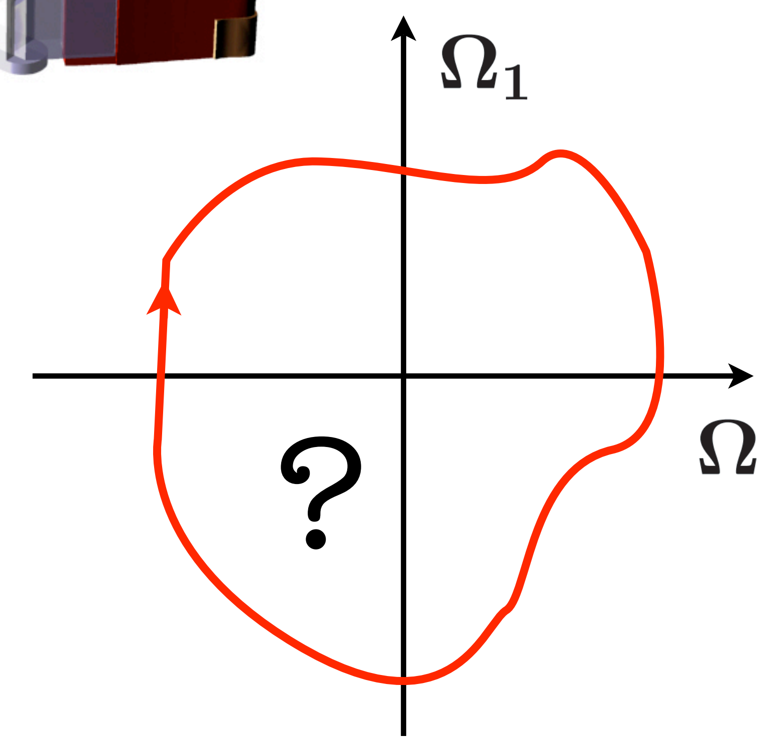
Can we do better?



Optimising Kinematics

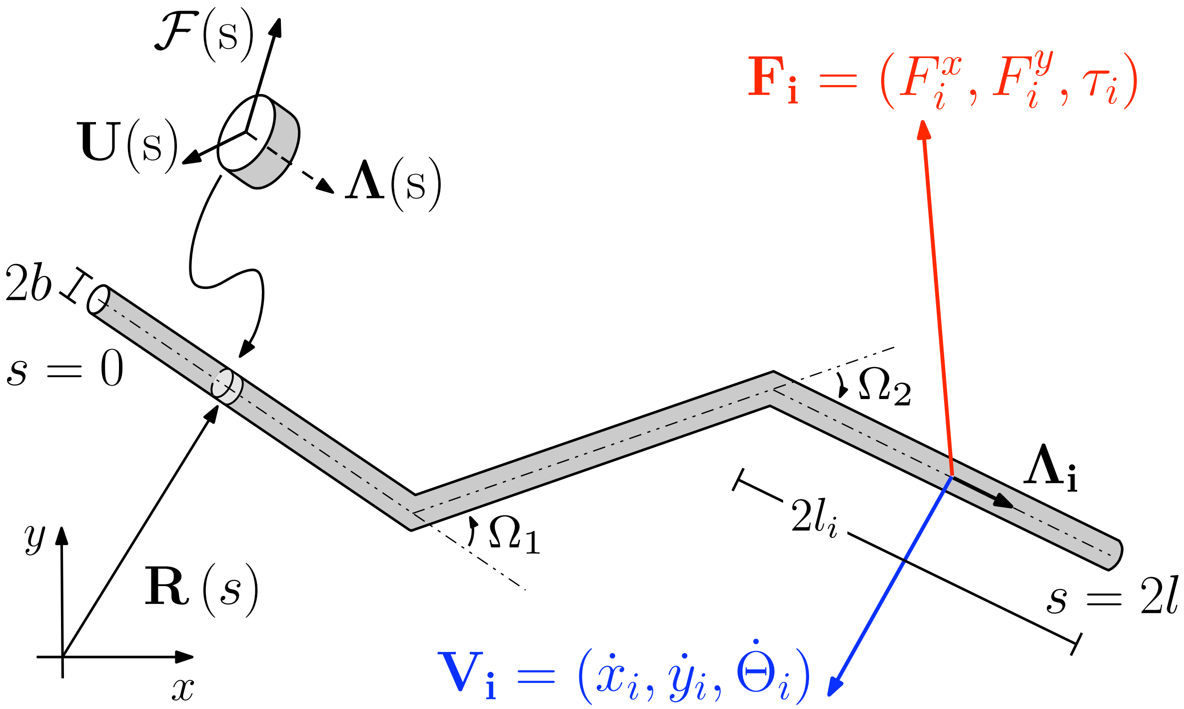
Fixed geometry

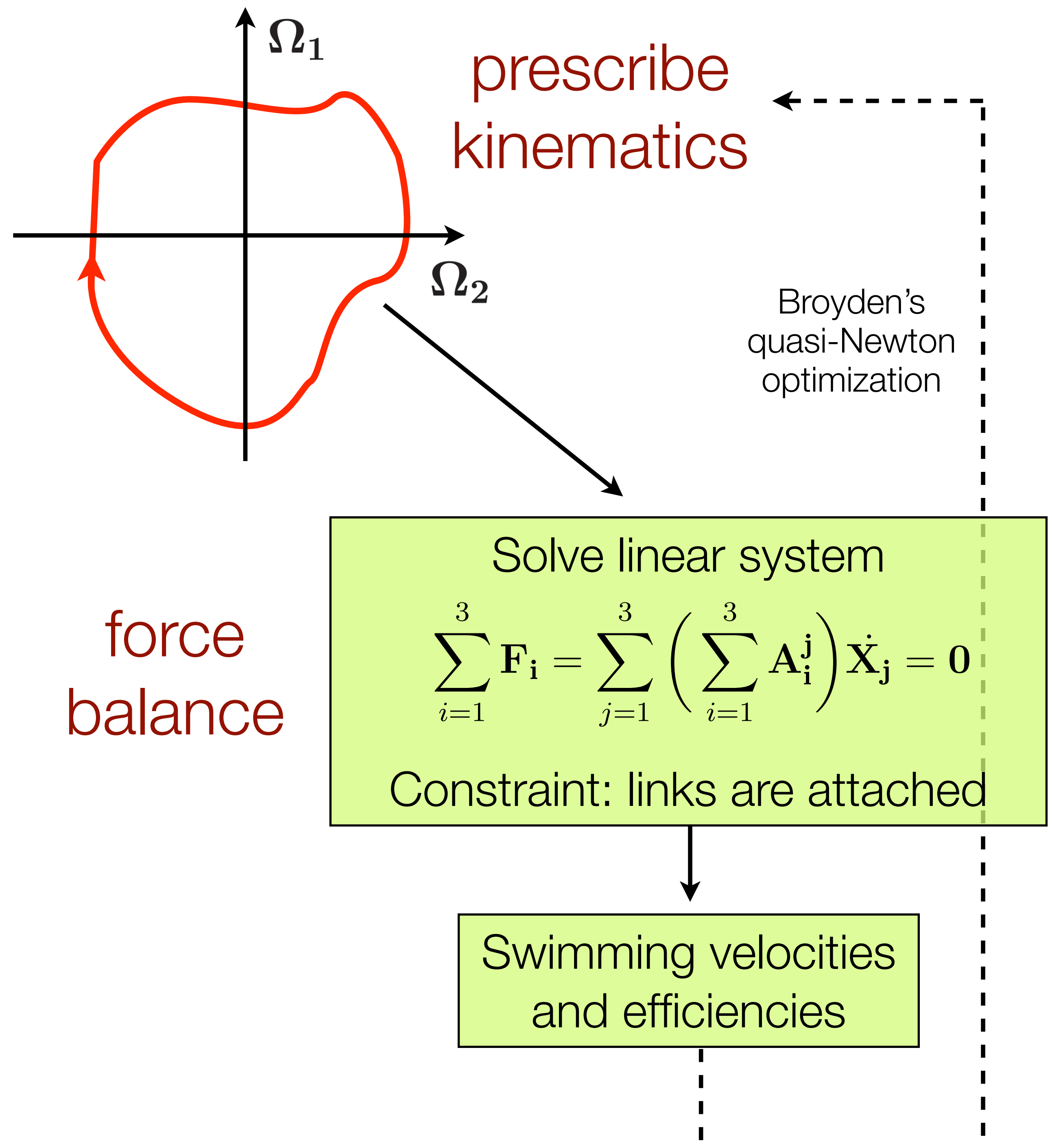




Kanso and Marsden (2005) - 3-link fish
Berman and Wang (2006) - insect flight

Model Swimmer

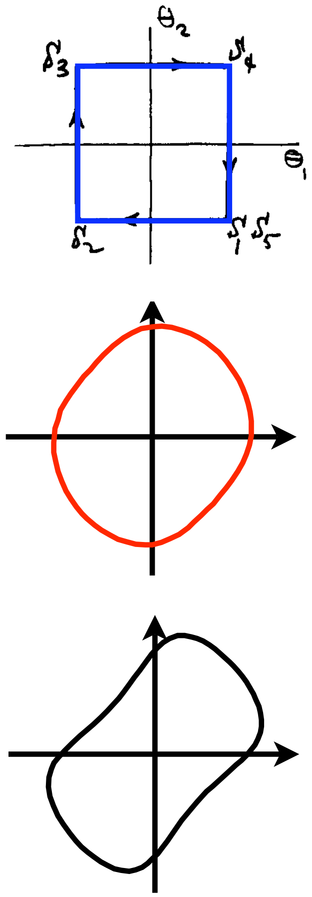


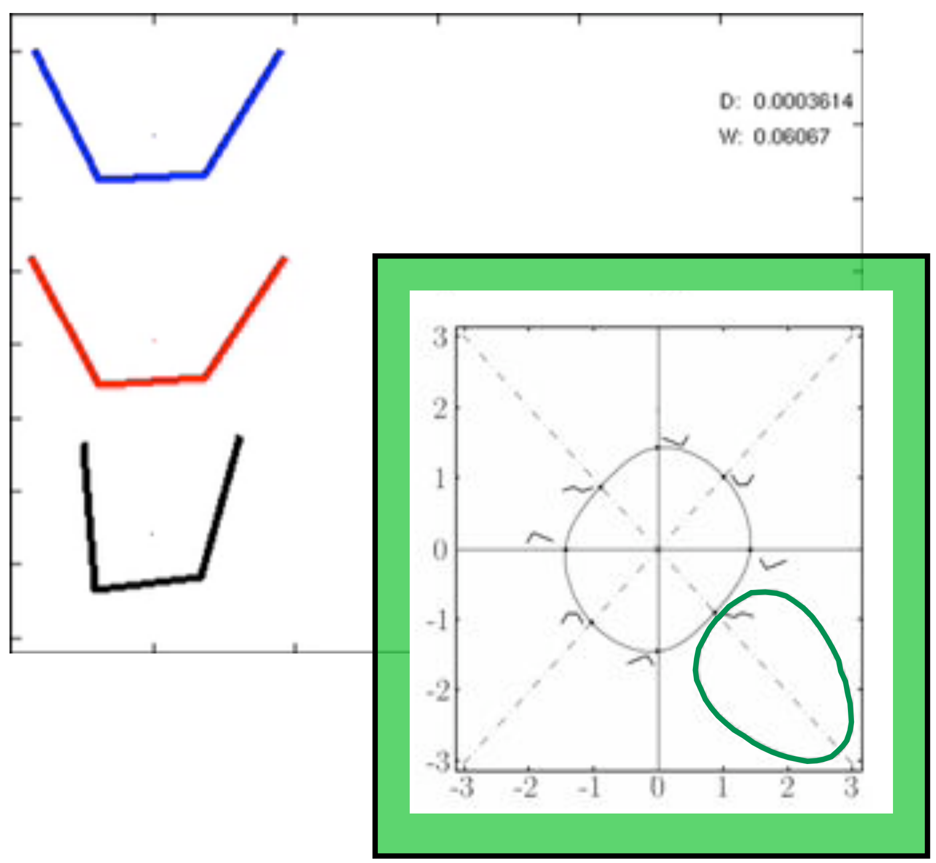


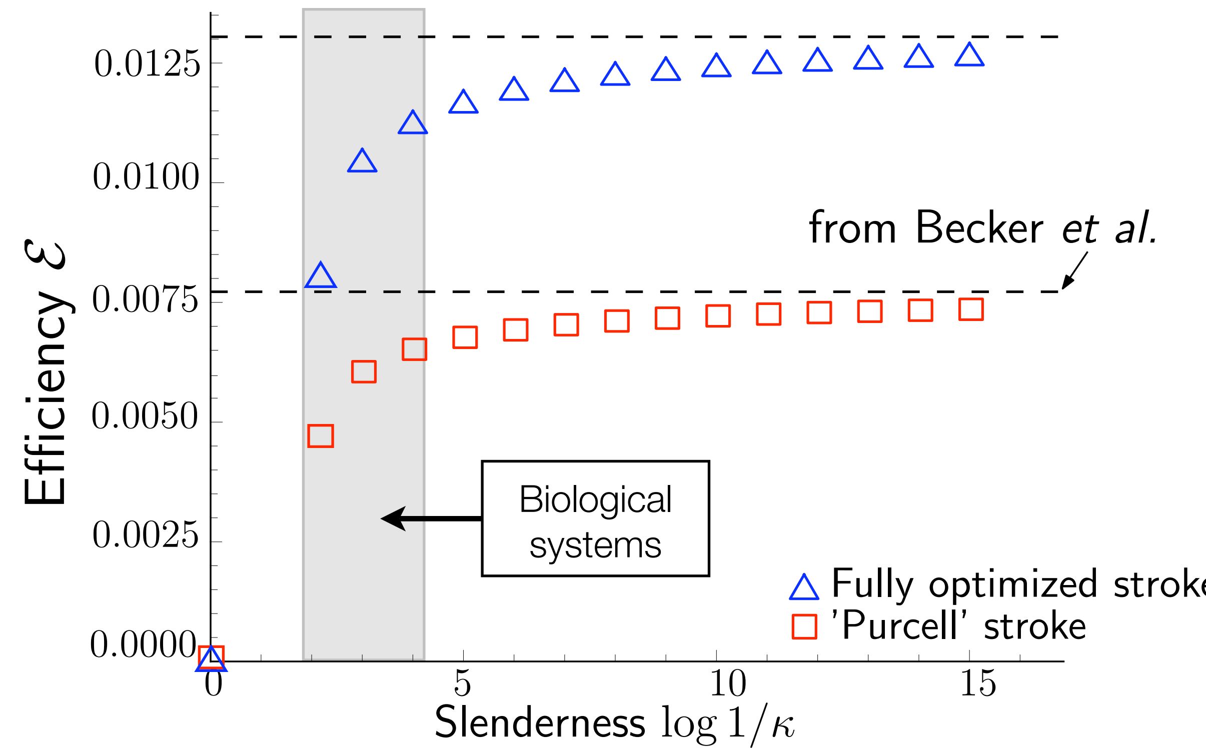
- Lowest order: resistive force theory
- Next order: can incorporate effects of slenderness and interactions between links

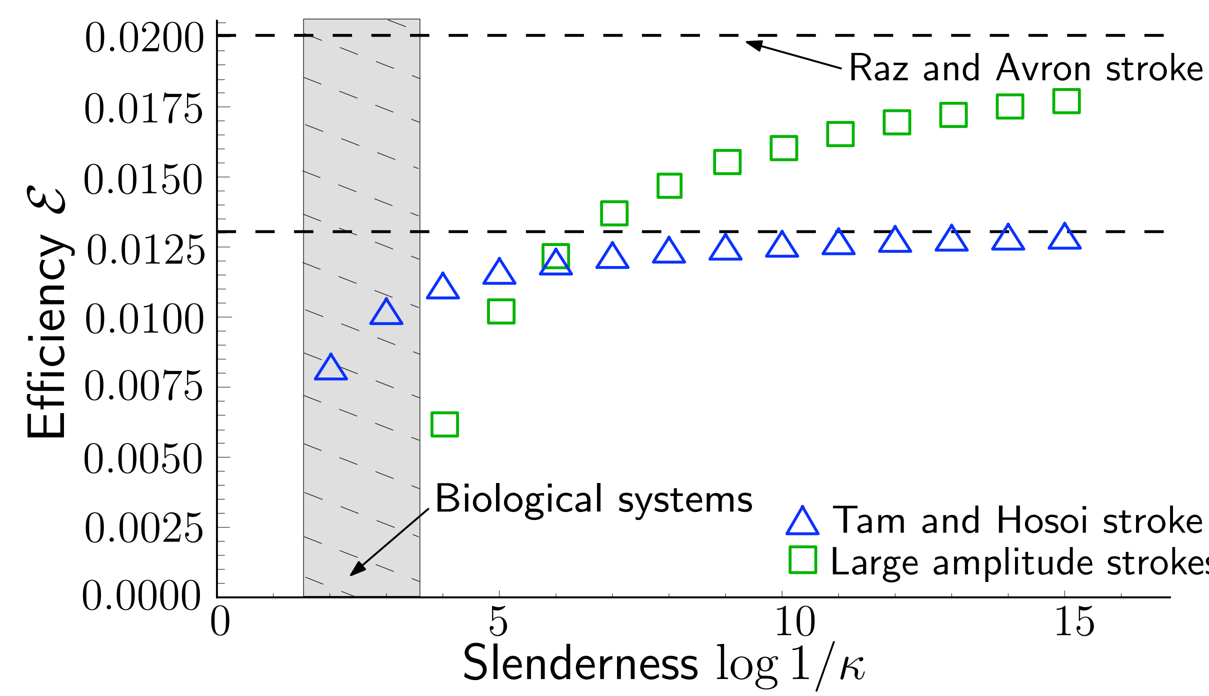
R. Cox. *J. Fluid Mech.* **44** (4), 791 (1970).
J. Keller and S. Rubinow. *J Fluid Mech.* **75** 705 (1976)

Effect of Slenderness





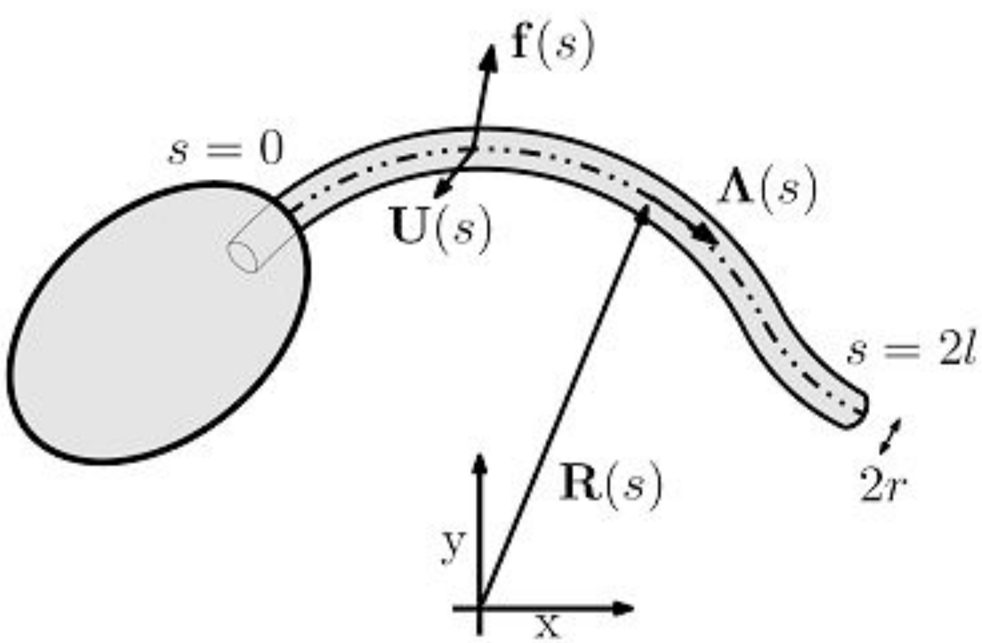




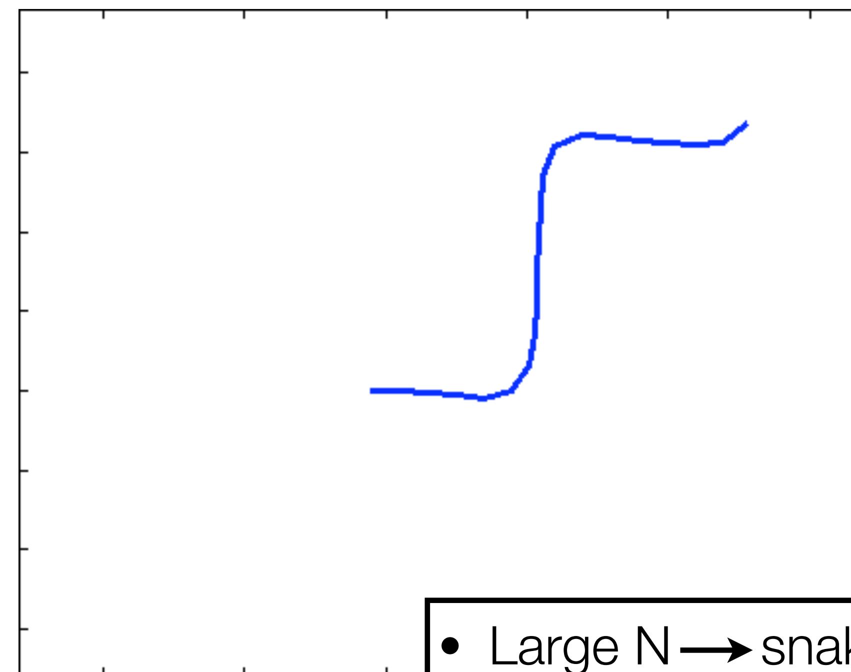
- Optimise kinematics for each value of slenderness more slender is better.
- Biological systems sit at the “knee” (trade-off between robustness and efficiency)
- Raz and Avron found more efficient large amplitude strokes
  - only more efficient for very slender flagella (~3 OM larger than those found in nature)

Kinematics of uniflagellates

- Flagellum: Slenderbody theory - find Stokeslet distribution (Keller and Rubinow, 1976)
- Head: Exact singularity distribution (Chwang and Wu. 1974)
- Head flagellum interaction: Faxen's laws (Happel and Brenner)
- Find optimal curvature along the tail



No head:

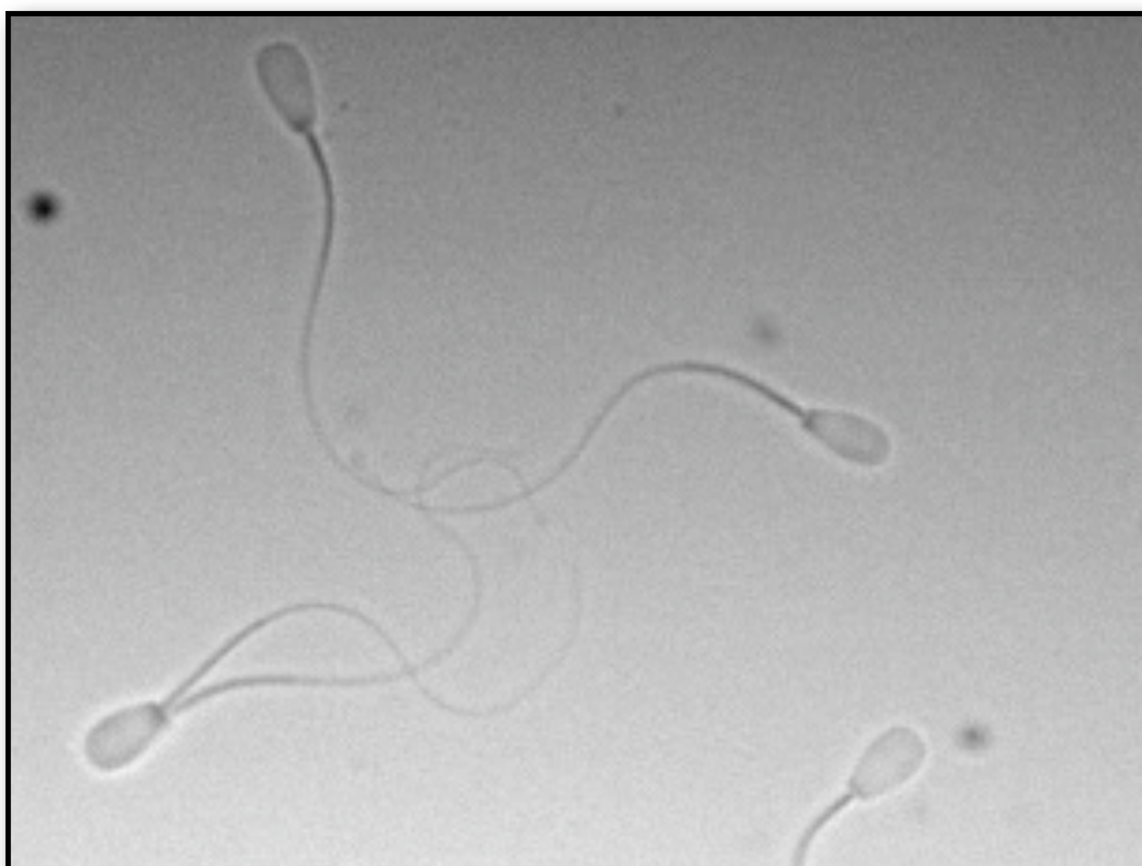


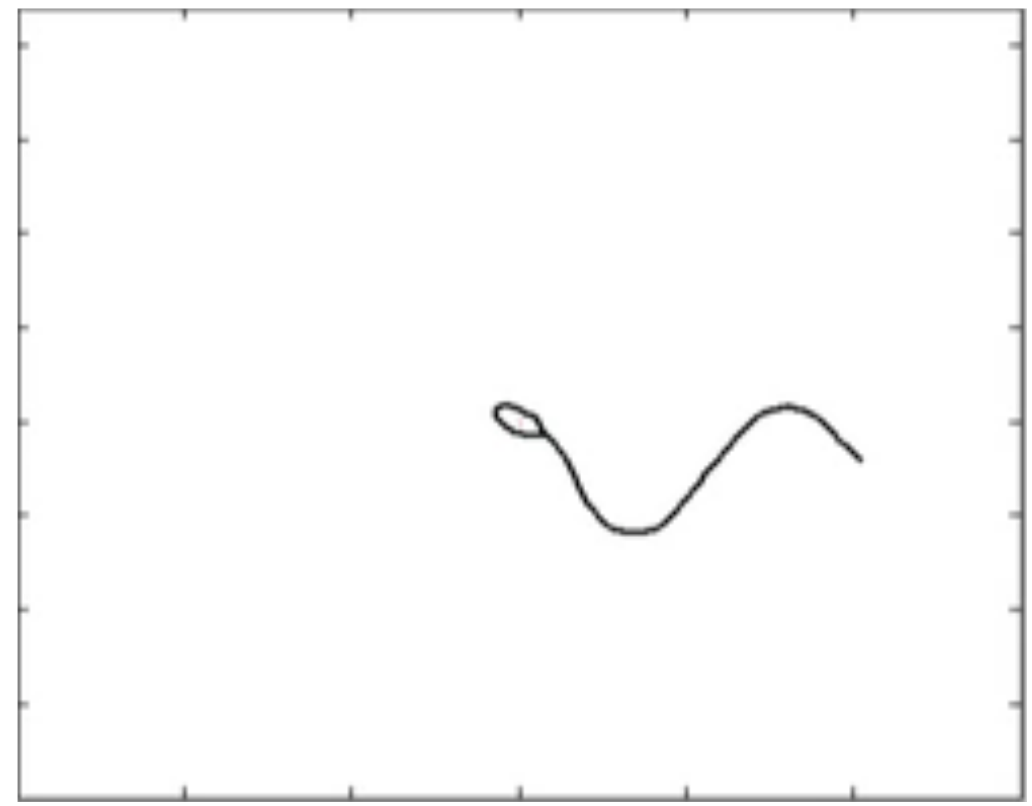
- Large N ⟶ snake
- Analytic solution (in Lighthill's *Mathematical Biofluiddynamics*)
- 41 degree angle

	Ψ	$\mathcal{E}$	U/V
Analytical solution	40°	0.0857	0.29
Computed solution	$\sim 41^\circ$	~ 0.08	~ 0.25

Kinematics of uniflagellates

- Travelling wave (~ one wavelength)
- Localized regions of high curvature connected by segments of ~ zero curvature
- Curvature decreases from head to tail

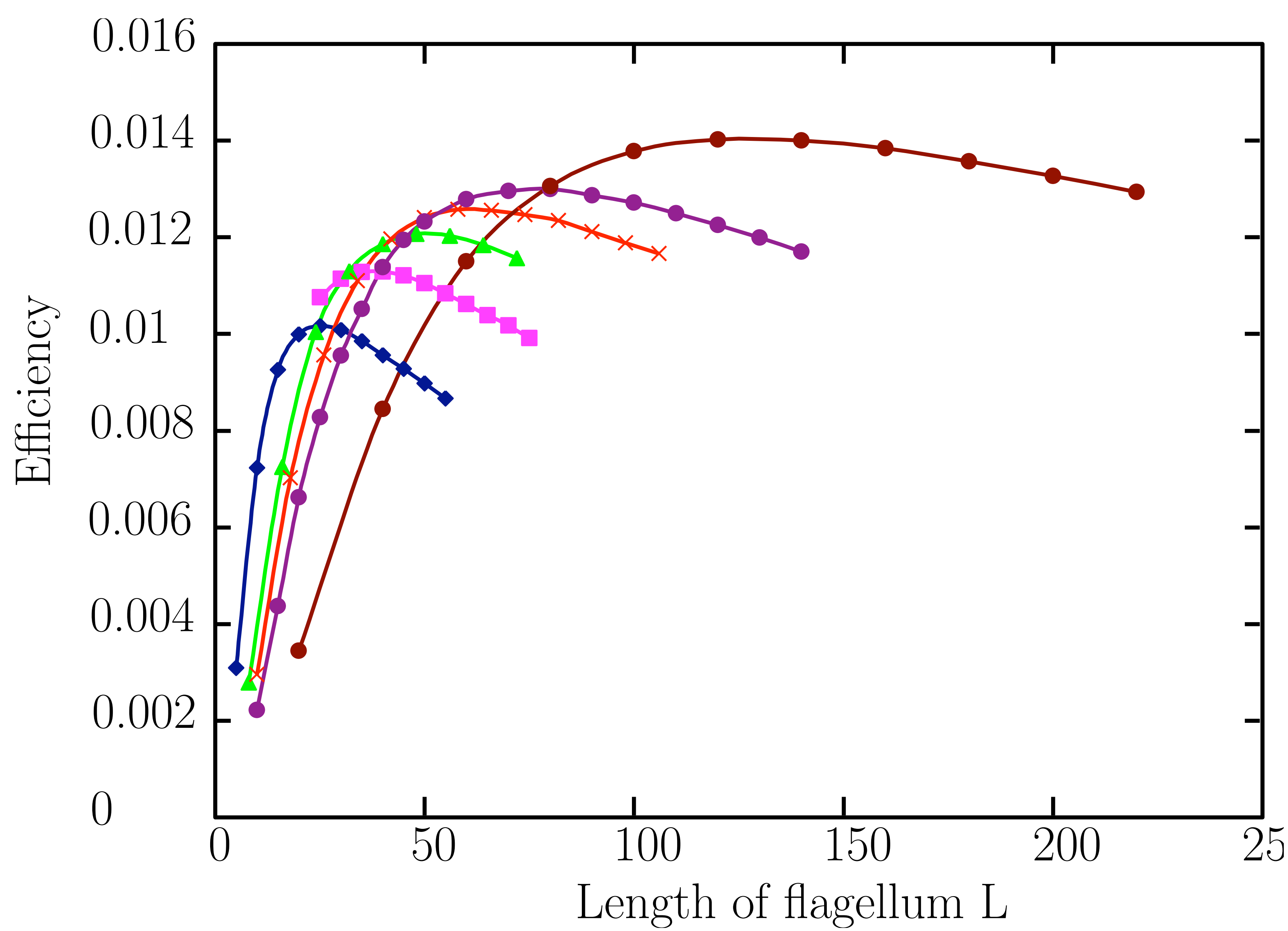




Optimal Tail Length

Goal: To move genetic material

Q: For a given head size, what is the optimal tail length?



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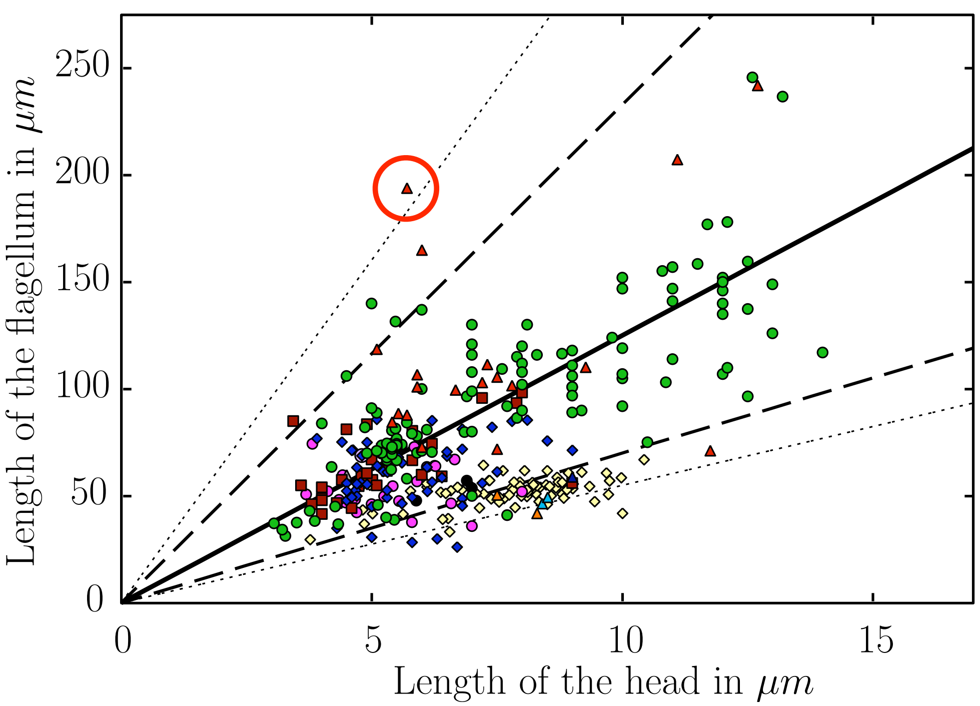
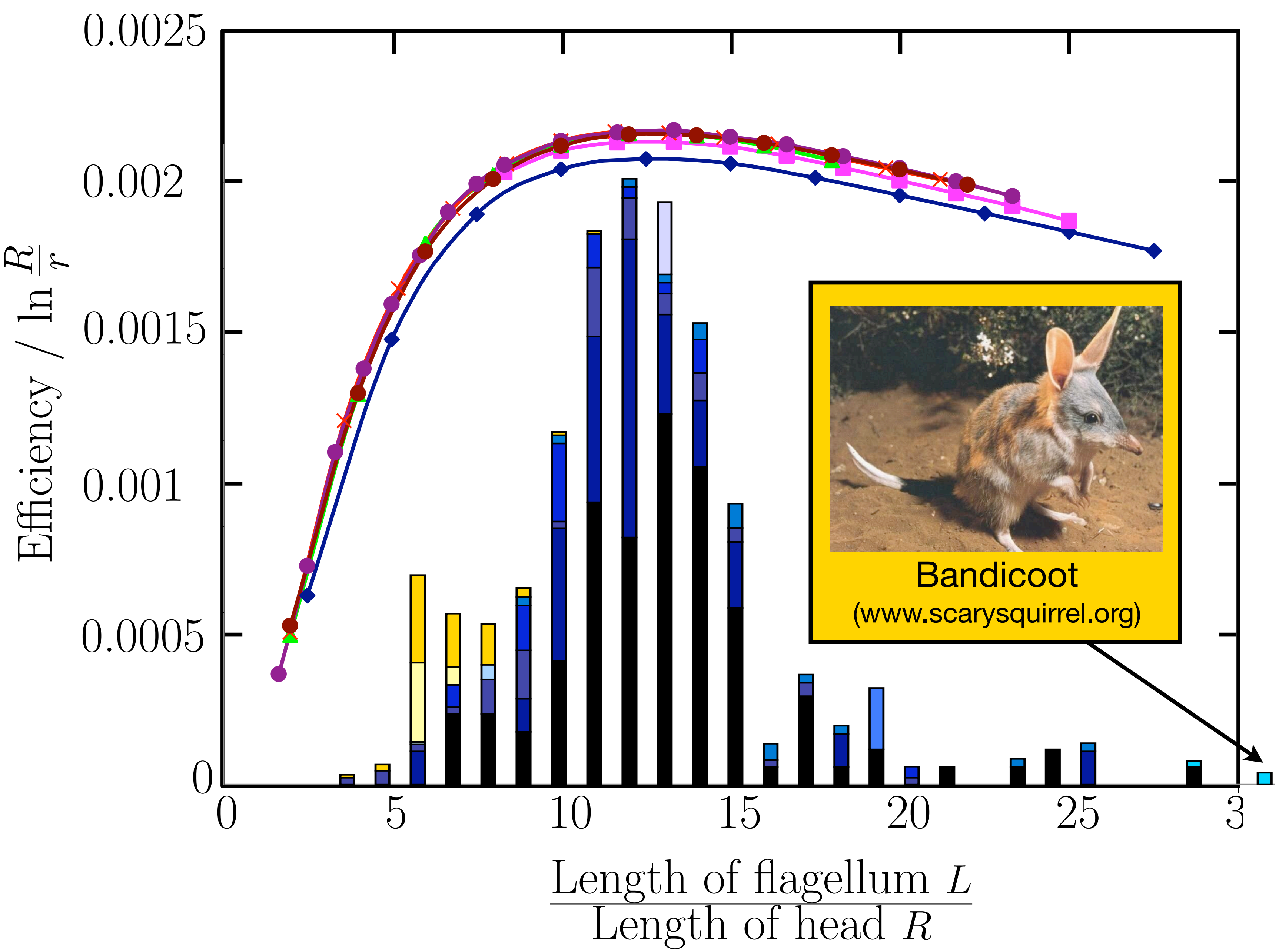
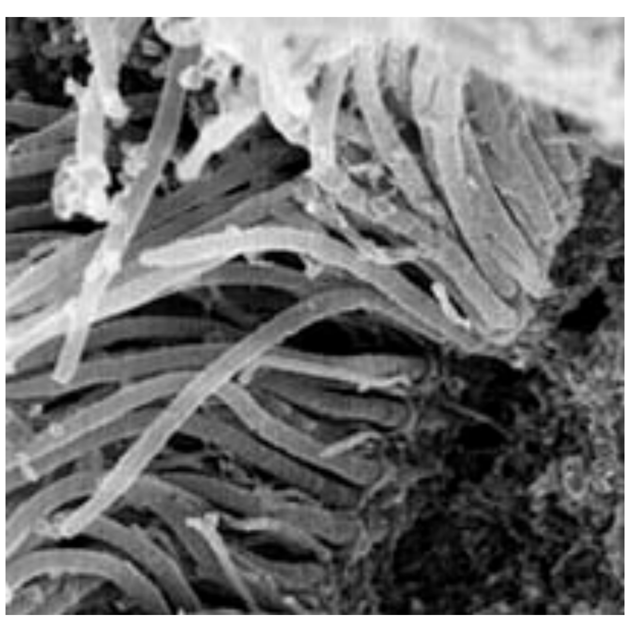
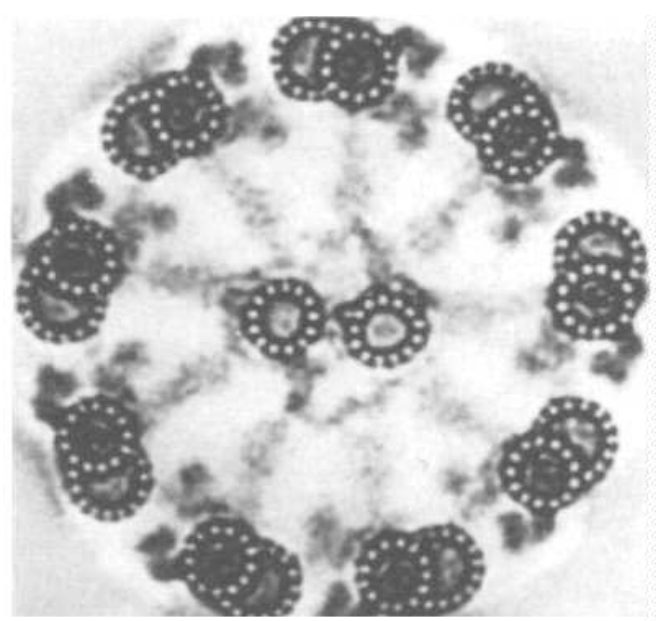


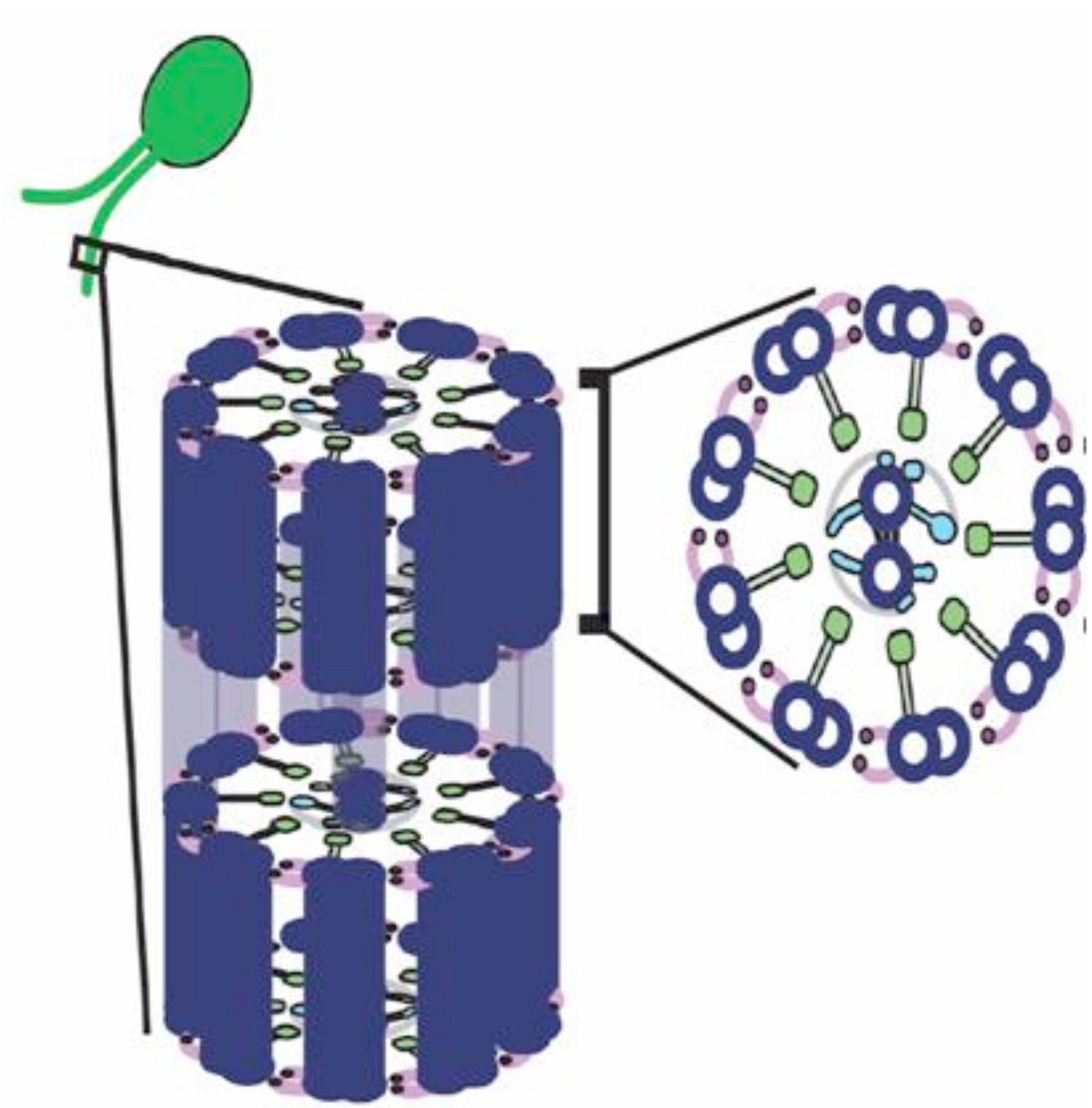


Structure of Flagella and Cilia

- Eukaryotic cells (flagella and cilia)
  - 9+2 microtubule structure
  - Diameter of tail ≈ 250-400 nm ≈ constant across ALL species!
  - Organism can apply local bending moments along the tail ➞ can select shape as a function of time (control kinematics)







Wemmer & Marshall, 2004

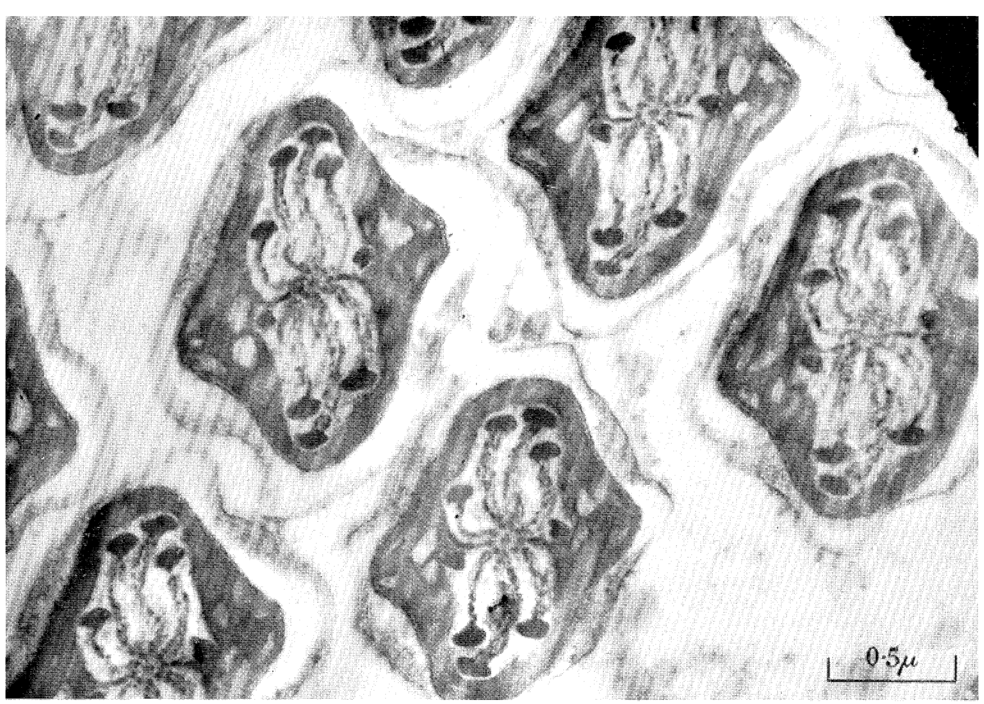
http://sun.menloschool.org/~cweaver/cells/e/cilia_flagella/
http://cellbio.utmb.edu/cellbio/cilia.htm

Structure of Flagella and Cilia

Bandicoot
(www.scarysquirrel.org)

- Eukaryotic cells (flagella and cilia)
  - 9+2 microtubule structure
  - ~~Diameter of tail ≈ 250-400 nm ≈ constant across ALL species!~~

Diameter of tail is approximately constant across all species EXCEPT bandicoots.

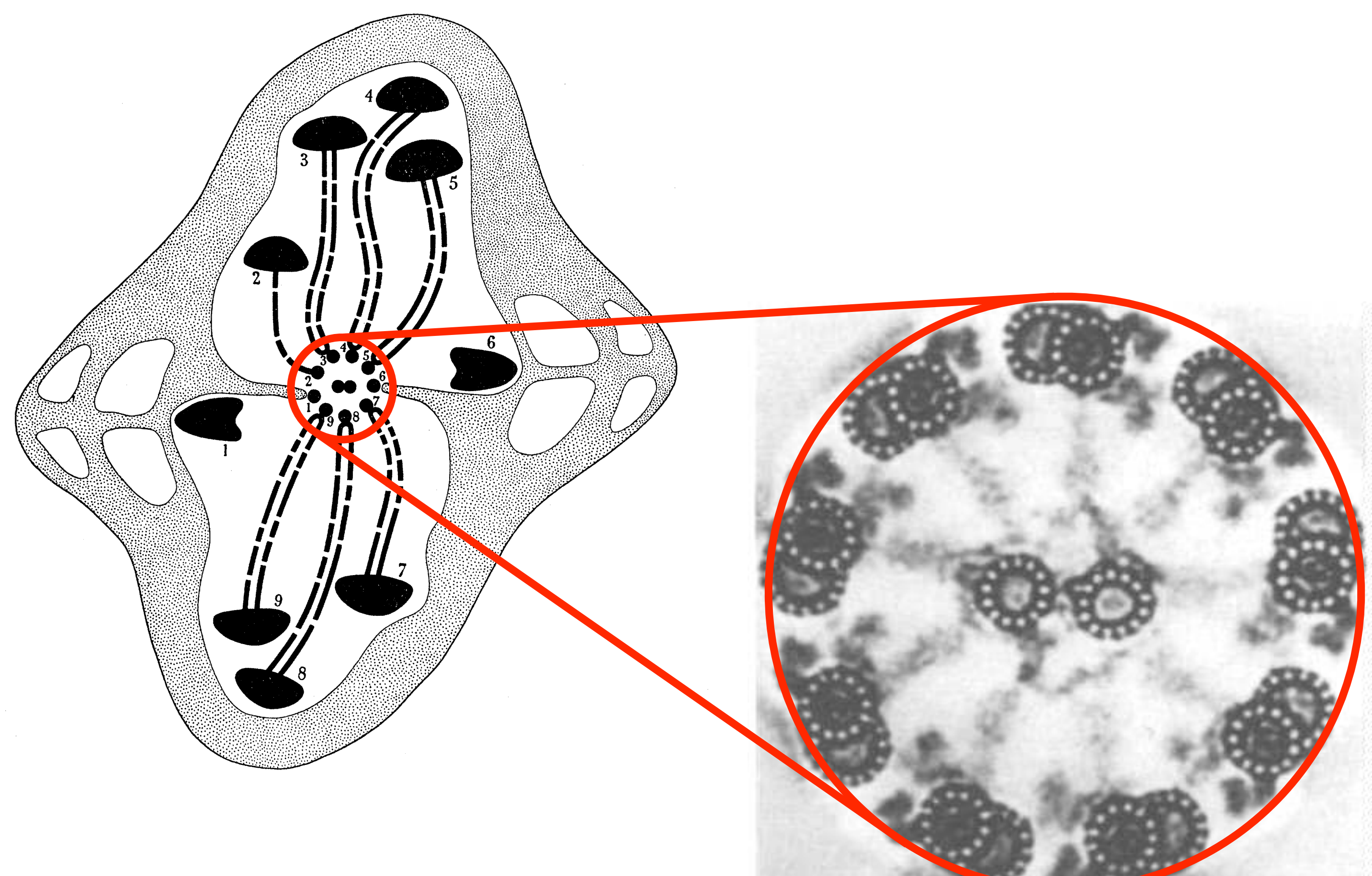


The bandicoot spermatozoon: an electron microscope study of the tail

By K. W. Cleland and Lord Rothschild, F.R.S.

Department of Histology and Embryology, University of Sydney, Australia, and Department of Zoology, University of Cambridge

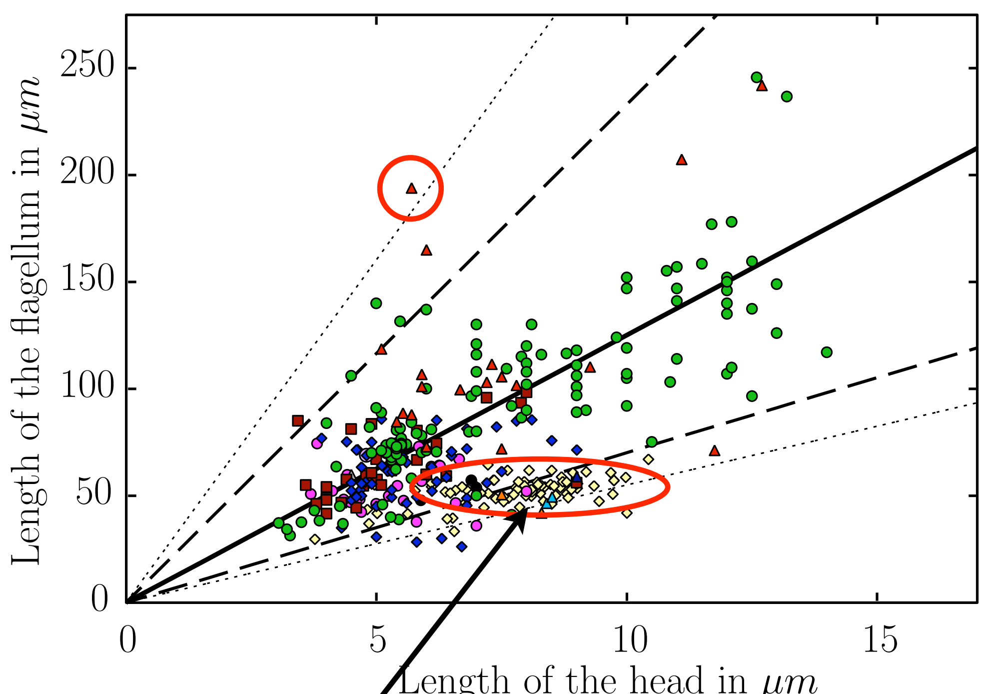
(*Received* 15 *July* 1958)



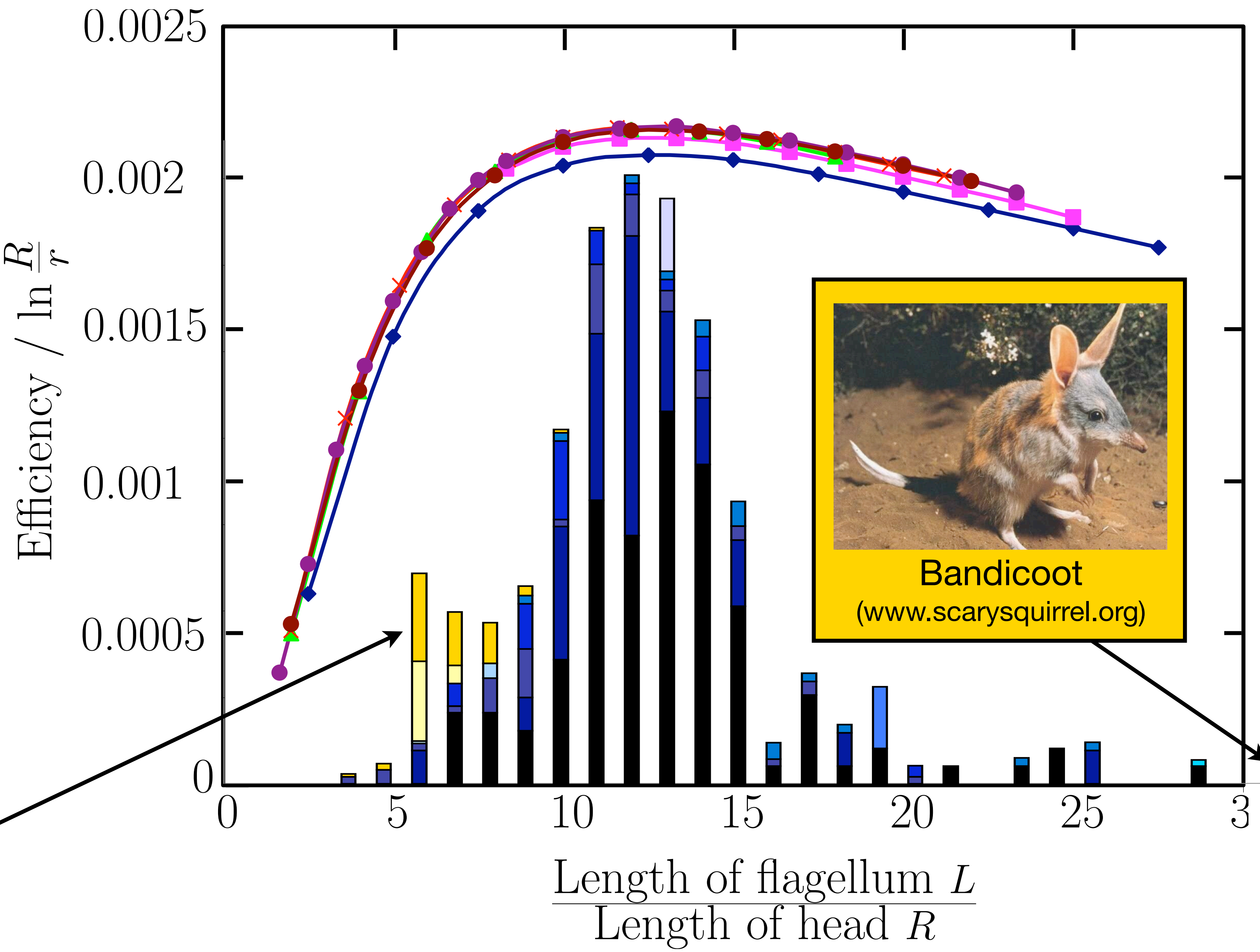
Optimal Tail Length

Goal: To move genetic material

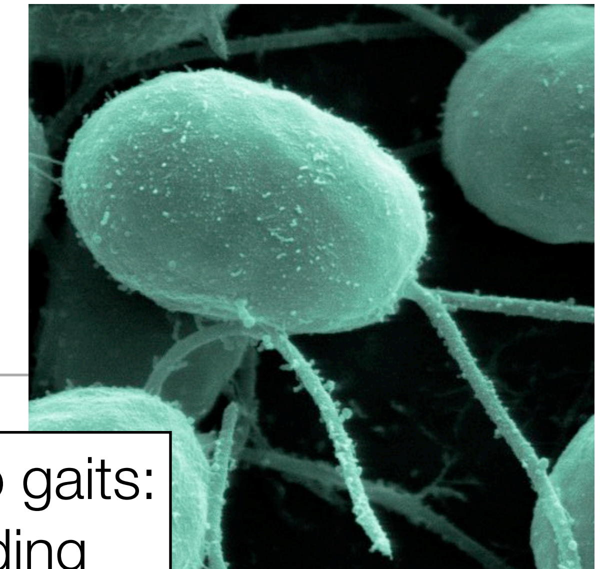
Q: For a given head size, what is the optimal tail length?



Order Artiodactyla
(even-toed ungulates)

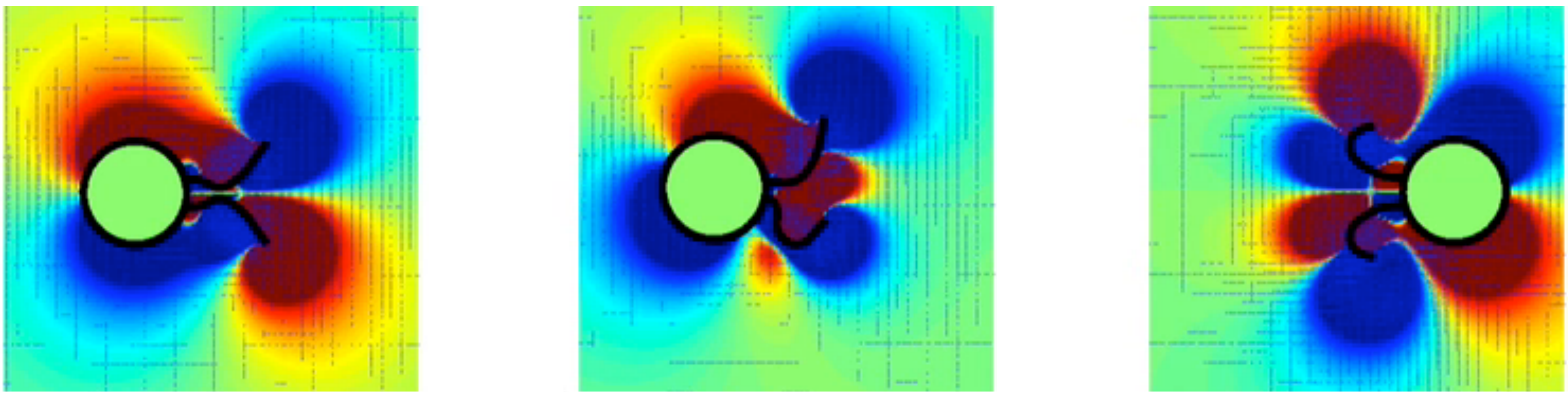


Biflagellate Kinematics



Goal: Enhance nutrient uptake
OR
Out-run predators

Expect to see two gaits:
- "Normal", feeding
- Escaping



Escaping — Feeding

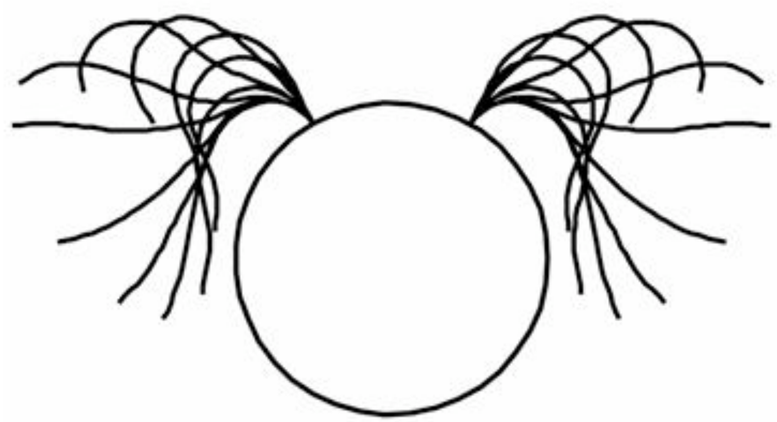
- Same objective function as uniflagellates
  - Traveling waves (two sperm tails)
- More complex optimization space - multiple local maxima

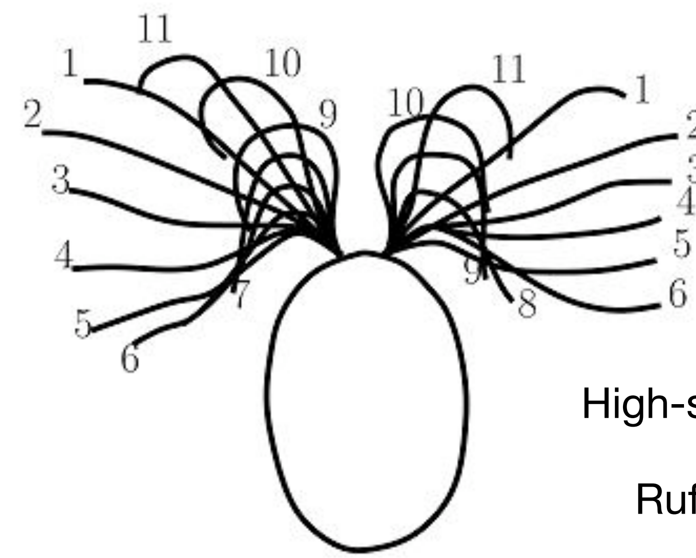
- Breast stroke (effective/recovery)

Compare with Biology

From Ken Foster's and Juree Saranak's homepage

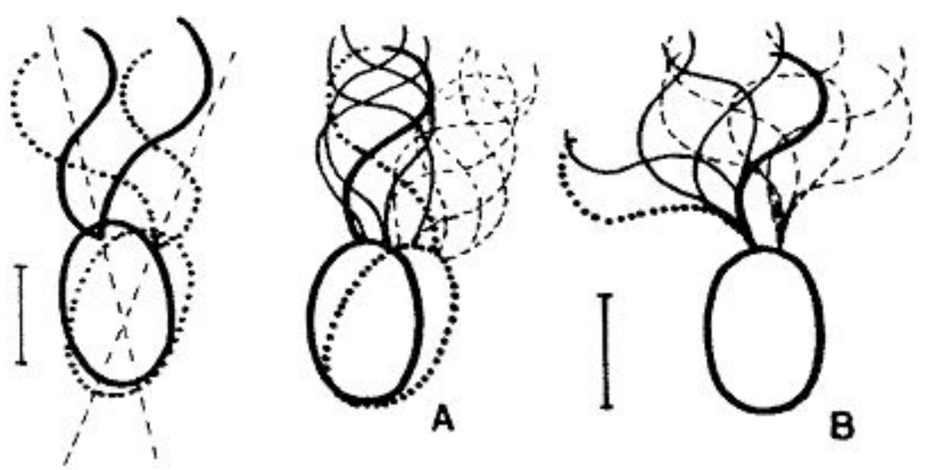
- Two commonly observed beat patterns
- "Normal" swimming - effective/recovery stroke (breast stroke)





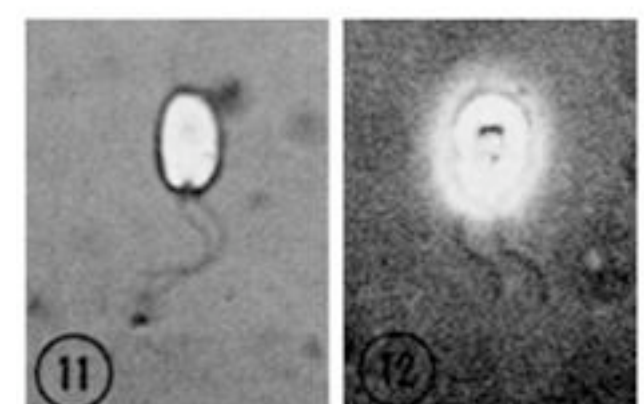
High-speed cinematography of
chlamydomonas
Ruffer and Nultsch (1985)

- Escape (shock response) - "hula" (traveling wave)



High-speed cinematography of chlamydomonas
Reinhardtii stroke, Rüffer and Nultsch (1998)

"Waves of bending, probably traveling from base to tip, pass along the flagella and exert a pushing force." D. Ringo (1967)

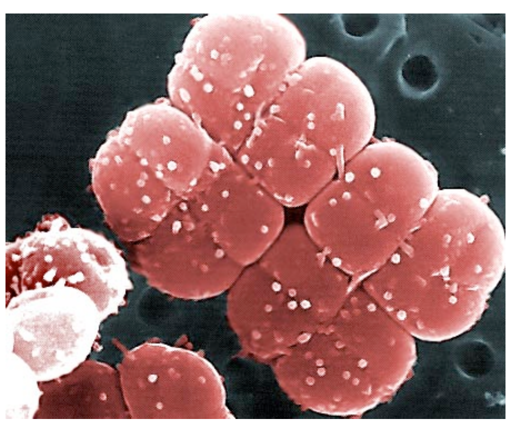


Acknowledgments

Real work done by DANIEL TAM

Thanks to Dr. Linda Turner (Harvard) and Prof. Susan Suarez (Cornell)
for many many many biology lessons

Funding by NSF



D. radiodurans (the world's toughest bacteria)

Back-up Slides

Model Swimmer

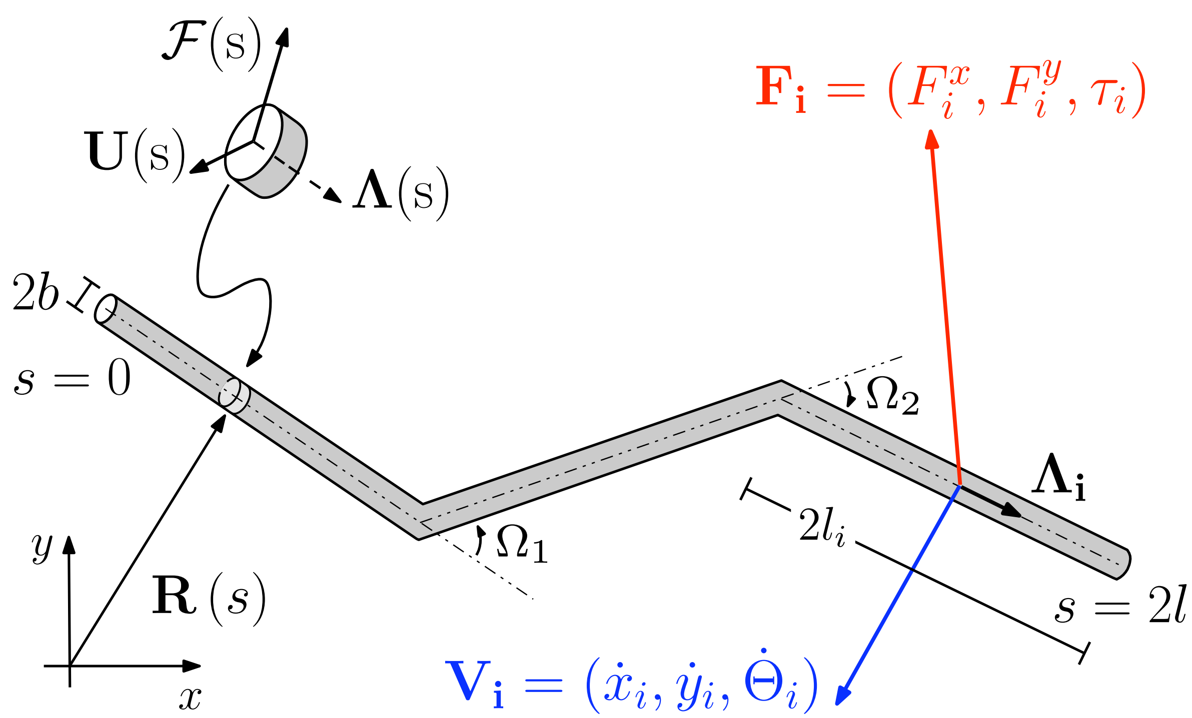
Force per unit length (slender body theory):

$$\frac{\mathcal{F}}{2\pi\mu} = \left[\frac{-\mathbf{U}}{\ln\kappa} + \frac{\lim_{\epsilon\to 0}\,(\mathbf{J}-\mathbf{U}\ \ln(2\epsilon))}{(\ln\kappa)^2}\right]\cdot[\mathbf{\Lambda\Lambda}-2\ \mathbf{I}] + \frac{-\mathbf{U}}{2(\ln\kappa)^2}\cdot[3\mathbf{\Lambda\Lambda}-2\ \mathbf{I}]$$

$$\mathbf{J} = -\frac{1}{2}\left[\int_0^{s-\epsilon} + \int_{s+\epsilon}^{2l}\right]\left[\frac{\mathbf{I}}{|\mathbf{\Delta}|} + \frac{\mathbf{\Delta\Delta}}{|\mathbf{\Delta}|^3}\right]\times\left[\mathbf{I}-\frac{1}{2}\hat{\mathbf{\Lambda}}\hat{\mathbf{\Lambda}}\right]\cdot\hat{\mathbf{U}}\,d\hat{s}$$

$$\mathbf{\Delta} \equiv \mathbf{R} - \hat{\mathbf{R}}$$

Cox (1970)



R. Cox, Journal of fluid mechanics **44 (4)**, 791 (1970)

Force per link:

$$\mathbf{F_i} = \int_{2l_i}(\mathbf{f}\cdot\hat{\mathbf{x}}, \mathbf{f}\cdot\hat{\mathbf{y}}, \mathbf{R}\times\mathbf{f})ds = \sum_{j=1}^{3}\mathbf{A_i^j V_j}.$$

- Lowest order: resistive force theory
- Next order: can incorporate effects of slenderness and interactions between links