EFMC7 Manchester, September 15-18.

NUMERICAL SIMULATIONS OF HIGH-RAYLEIGH NUMBER THERMAL CONVECTION

R. Verzicco

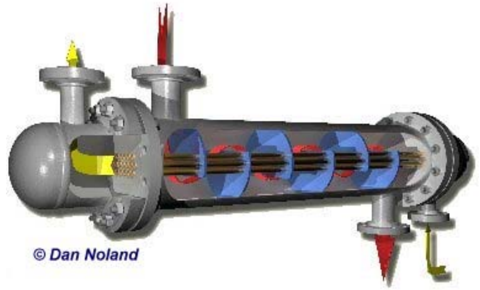
DIM Università di Roma "Tor Vergata", Roma, Italia.

- Thanks to:
- K.R. Sreenivasan, A. Sameen, G. Silano, ICTP Italy,
- G. Stringano, P. Oresta, Poliba Italy,
- K. Koal, G. Amati, F. Massaioli, CASPUR Italy,
- R. Camussi, Uniroma3, Italy.

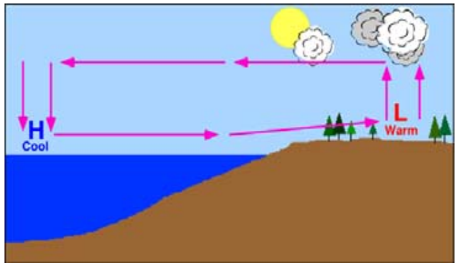
Motivation

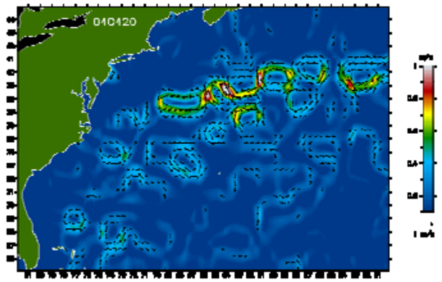
Heat transfer mediated by a fluid takes place in countless phenomena in industrial and natural systems, for example ....

....in cooling problems
(from CPUs to industrial plants)

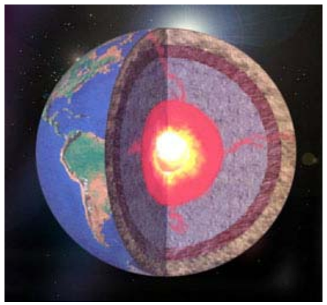


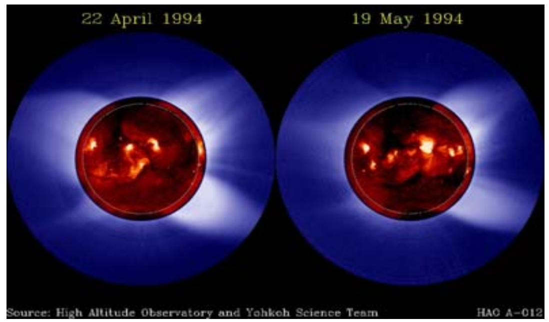
… in the motions of atmosphere and oceans driven by temperature differences





… in planets liquid core and stars convection



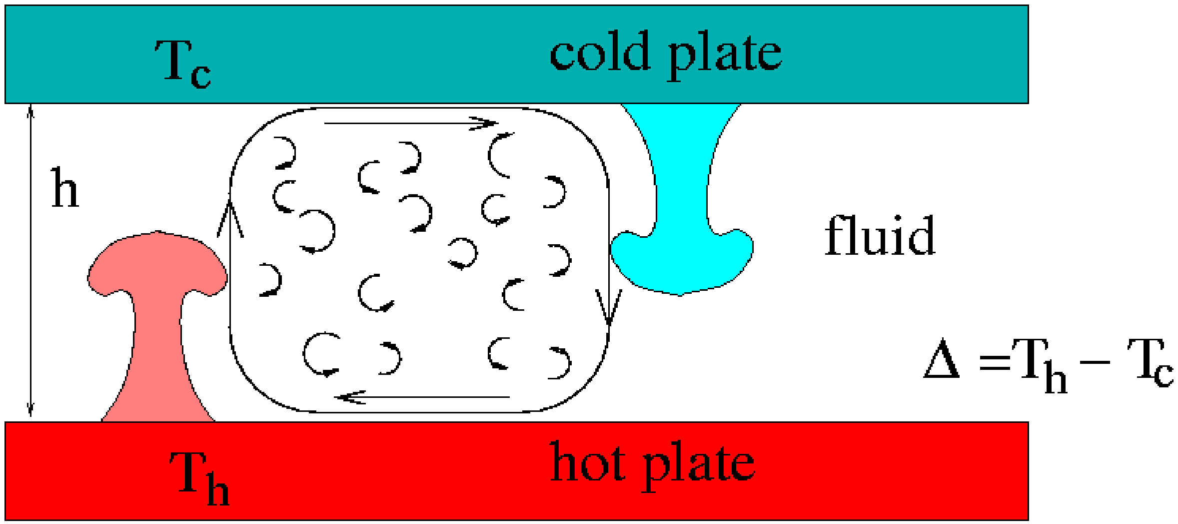


Interesting *per se* owing to rich and complex physics

The Rayleigh-Bénard problem

Fluid layer of depth *h* heated from below and cooled from above

Thermal expansion causes hot fluid to rise and cold fluid to sink
(unstable thermal stratification) *`only few exceptions'*



A flow is established if Δ exceeds a stability threshold

Rayleigh (1916), Benard (1900)

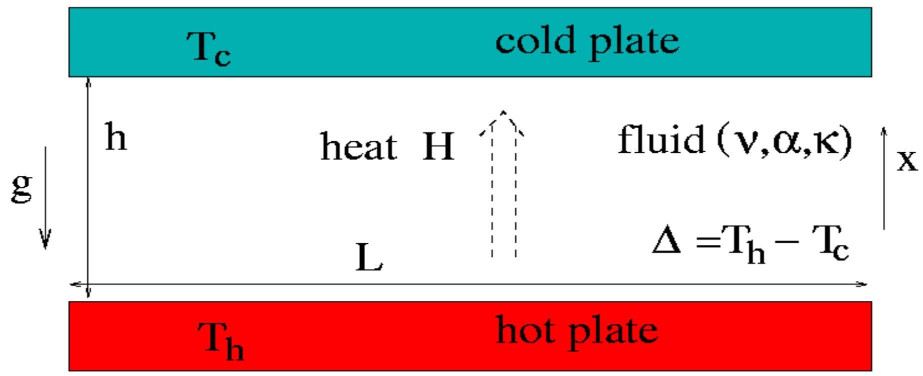
A model problem for countless practical applications

The Ideal Problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta\hat{x} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{D\theta}{Dt} = \frac{1}{(Pr\,Ra)^{\frac{1}{2}}} \nabla^2 \theta$$



$$Ra = \frac{g\alpha\Delta h^3}{\nu k}$$ `forcing' parameter

$$Nu = \frac{H\ h}{\lambda\ \Delta}$$

On input:

$$Pr = \frac{\nu}{k}$$ *fluid properties*

On output:

$$\mathrm{Re} = \frac{Uh}{\nu}$$

$$\Gamma = \frac{L}{h}$$ *geometric parameter*

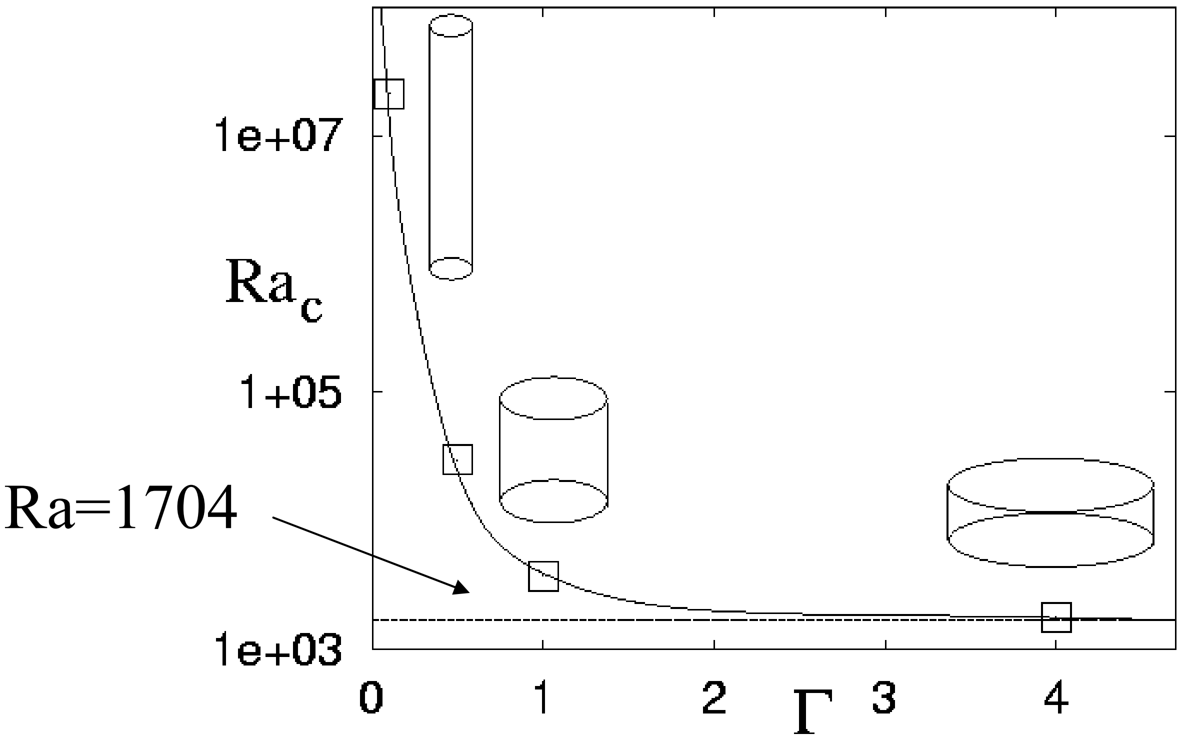
The Onset of Convection

The fluid starts moving only when $Ra > Ra_c$

(buoyancy must `exceed' viscous drag and heat diffusion)

Ra_c depends on boundary conditions and on the cell aspect ratio Γ

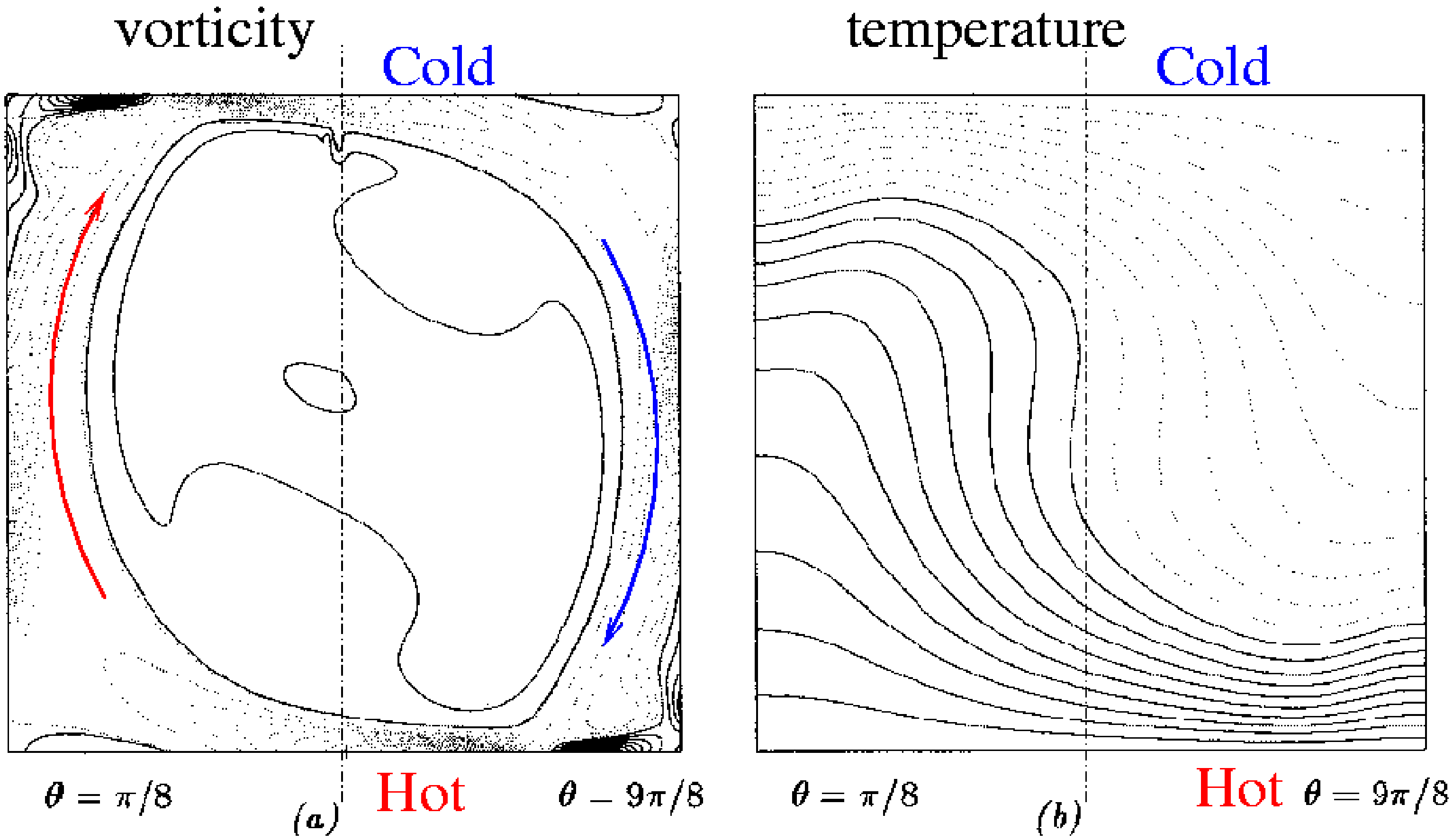
(Ra_c is independent of Pr)



The Onset of Convection

Steady smooth flow

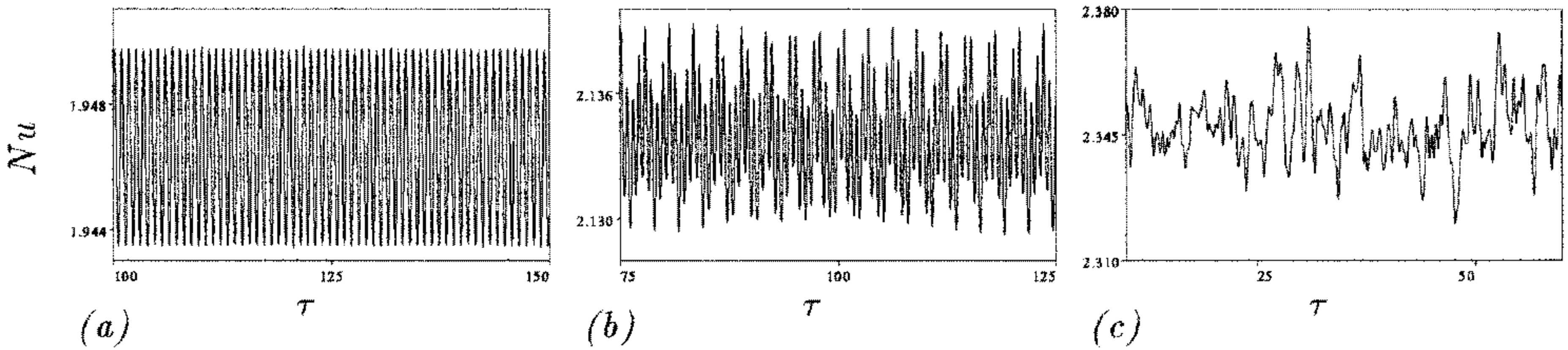
Weak dependence on the cell shape
(cylindrical, cubic, etc.)



Transitional Regimes

For increasing Ra the flow becomes time-periodic/multi-periodic chaotic and eventually turbulent

Dependence on Pr number, cell aspect-ratio Γ and cell shape

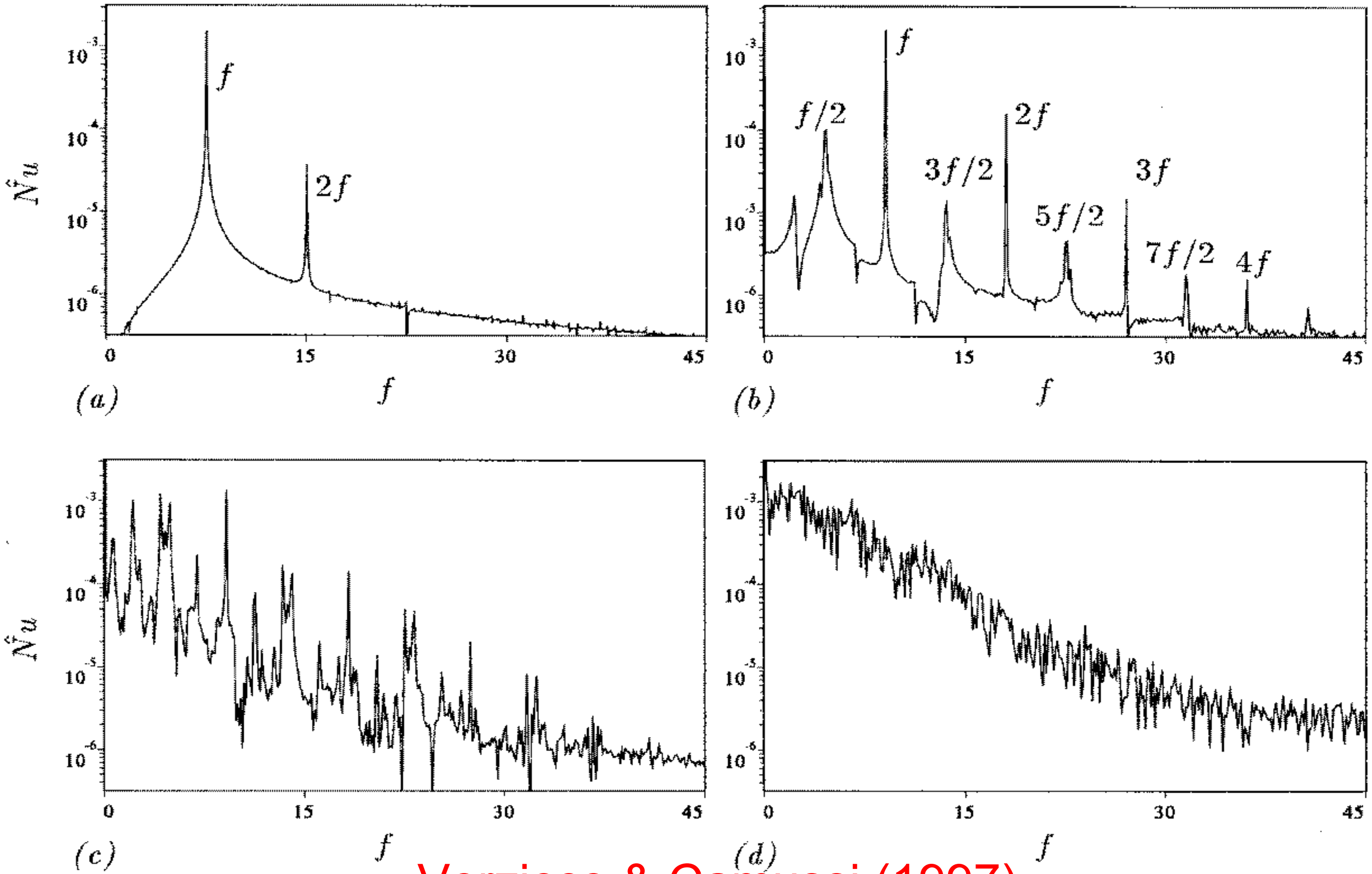


Verzicco & Camussi (1997)

Transitional Regimes

Transitions triggered by nonlinear terms by period-doubling and sub-harmonic mechanisms

A continuous spectrum indicates a turbulent flow

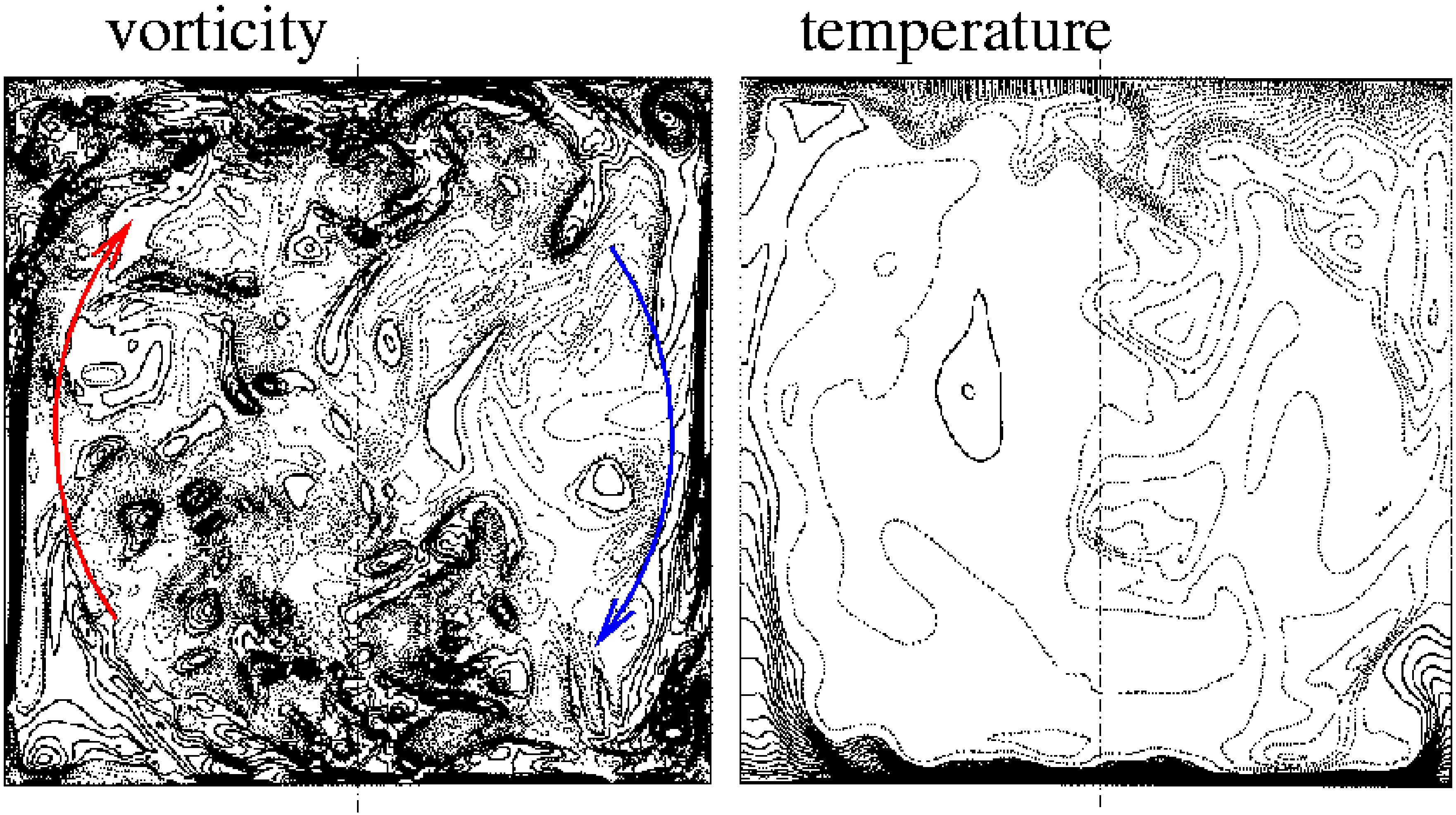


Verzicco & Camussi (1997)

Turbulent Regime

Thin viscous and thermal boundary layers

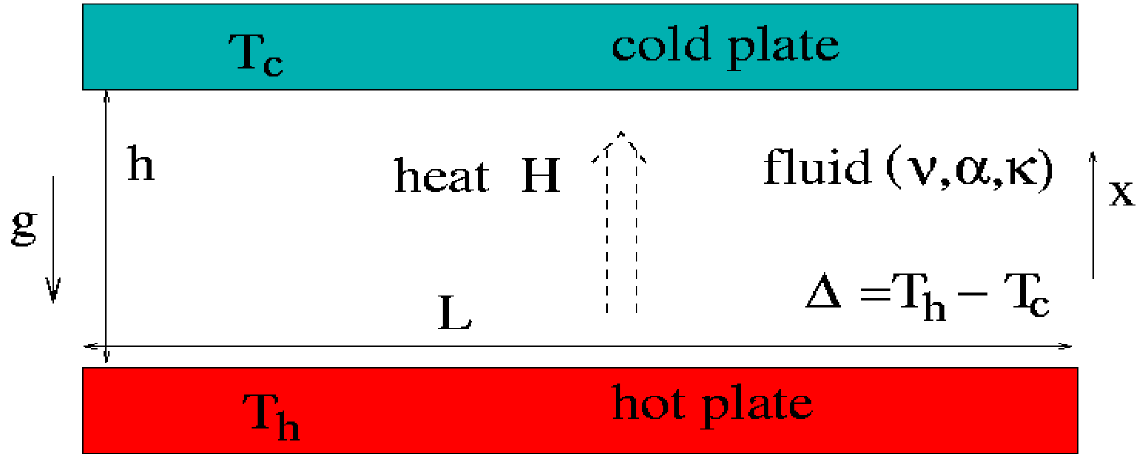
Small scales (in the bulk)



Problems at high-*Ra*

Main control parameter:
Rayleigh number

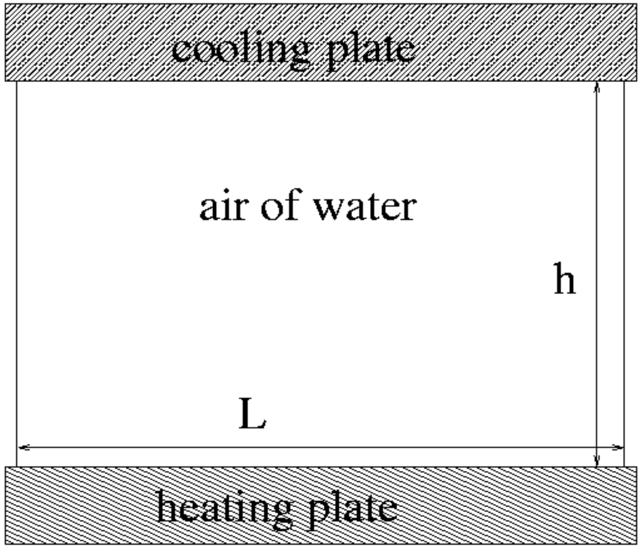
$$Ra = \frac{g\alpha\Delta h^3}{\nu k}$$



In real systems the Boussinesq approximation is often valid since Δ is limited to few degrees nevertheless Ra ≈ O(1.e6-1.e20) because of large system dimension [h ≈ O(1m-1.e4Km)]

Problem: how to reach high Ra in laboratory set-ups with h=O(1cm-1m)?

Typical Experiments



Practical considerations:

temperature homogeneity on the plates
total weight of the set-up
cost of the experiment

h=O(10-50 cm)

L/h=O(0.5-4)

For the Boussinesq approximation to hold: $\alpha\Delta \leq 0.1-0.15$

In air Δ < 30 K *(at ambient temperature)*

In water Δ < 20 K *(limited by other properties)*

$$Ra = \frac{g\alpha\Delta h^3}{\nu k}$$

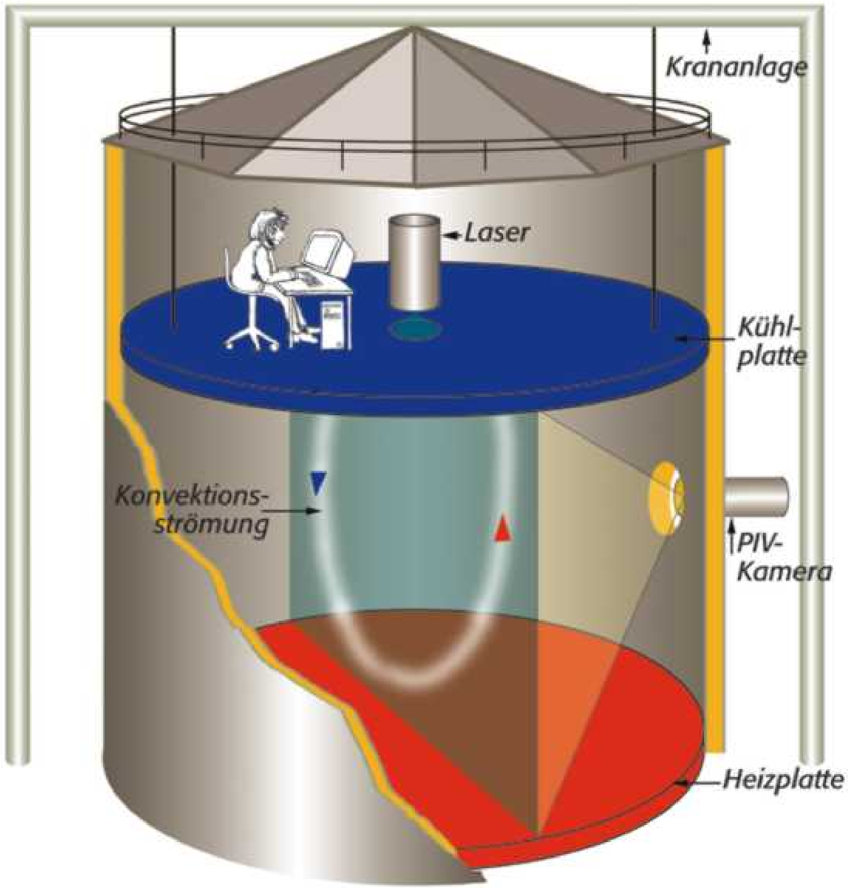
< 4.e+08 in air

< 2.e+10 in water

Extreme Experiments

Experiment	dimension	working fluid(s)	Ra_{max}
"Ilmeneau barrel"	h≈ 7m	air	10^{12}
Cost and controllability issues			
du Puits et al. (2007)			
liquid metals	h≈ 10-50cm	Hg, Na	$5x10^{11}$
Low Pr experiments: mercury vapour poisoning and explosive, liquid sodium high temperatures >350 °C			
Cioni et al. (1996), Takeshita et al. (1996), Rossby (1969)			
pressurized gasses	h≈ 10-50cm	N_2, Ar, SF_6	$5x10^{12}$
Very high pressures (up to 100 bar) large Pr variations			
Ashkenazi & Steinberg (1999) Fleischer & Goldstein (2002)			
cryogenic helium	h≈ 10-100cm	He at 4 K	$\approx 10^{17}$
Cryogenic temperatures, flow accessibility			
Chavanne et al. (2001), Niemela et al. (2000)			

The “Ilmenau Barrel”



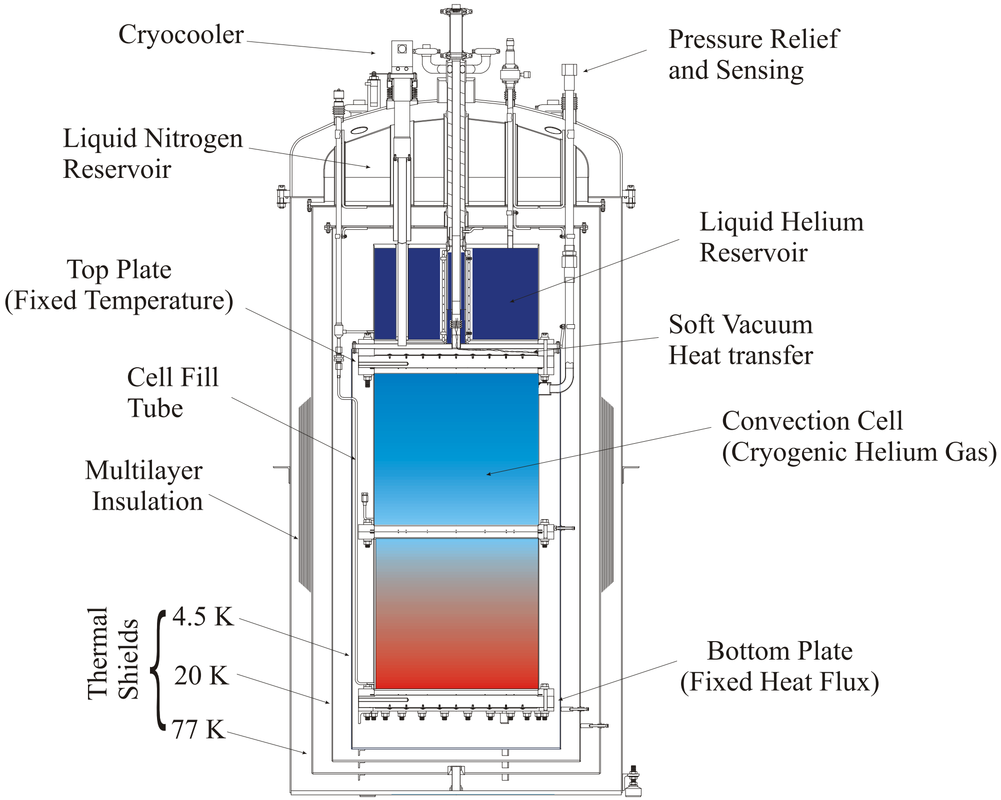
h≈ 7m L/h=1.1-11

Working fluid: air *(at ambient temperature)*

$$Ra = \frac{g\alpha\Delta h^3}{\nu k} \leq 10^{12}$$

A cryogenic apparatus for very high Ra (sample height = 1 meter, diameter = 0.5 meter)

Niemela et al. (2000)

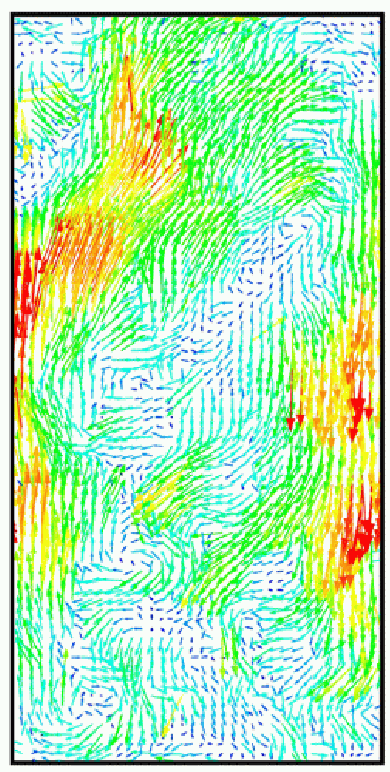


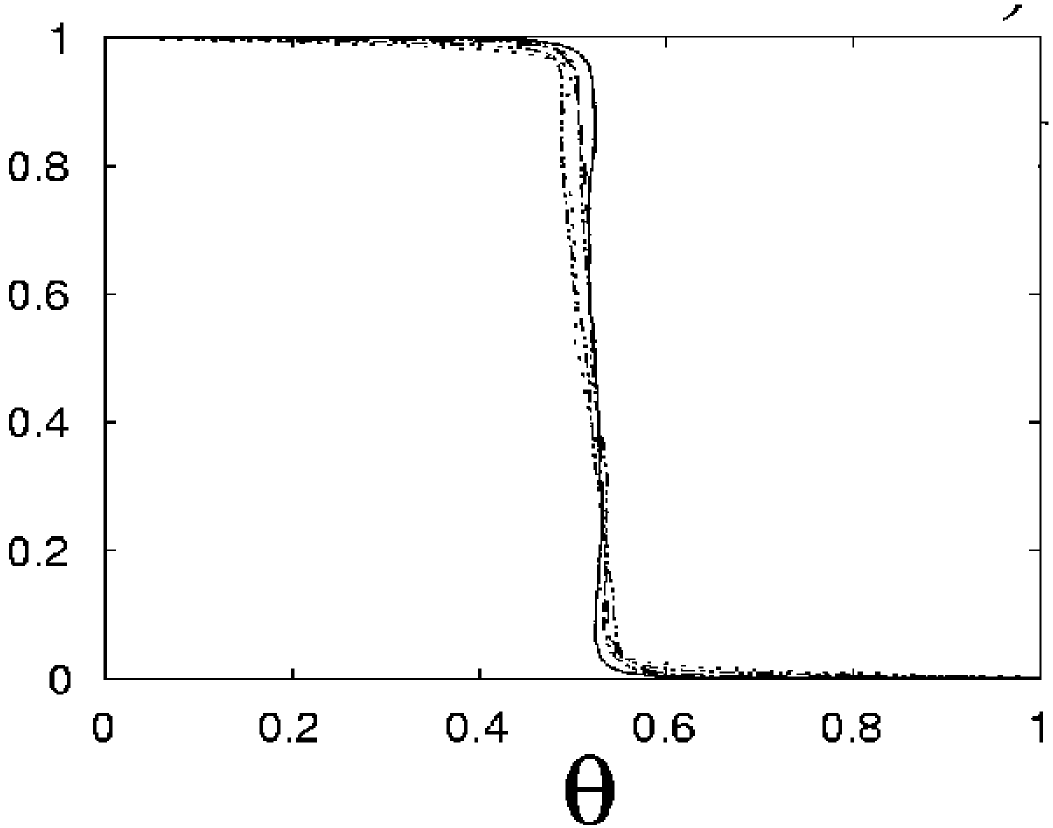
- Ra = $(g\alpha\Delta TH^3)/(\nu\kappa)$ ~ constant*$(\rho^2\alpha C_P)$. Ra increases as ρ^2 in ideal gas regime and as αC_P near critical point. αC_P is decades larger than for conventional fluids.

- 11 decades of Ra possible! Large sample height moves *entire range of Ra* into turbulent regime and indirectly extends conditions of constant Pr (ideal gas) to higher Ra.

Limitations of Laboratory Measurements

Most of density variation occurs within the thermal boundary layer:
PIV possible only in the bulk.





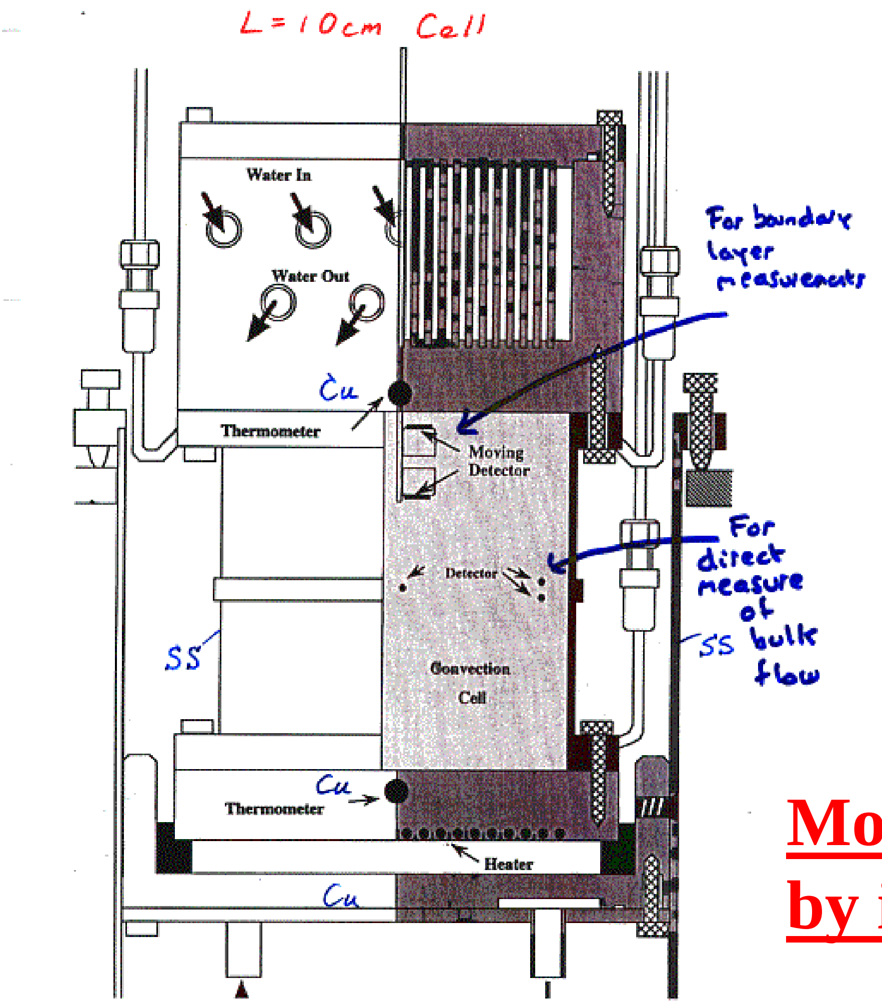
Practical consideration:
Thermal b.l. inaccessible by optical measurements

Sun & Xia (2005): PIV measurements

Kunnen et al. (2008): stereo PIV

Limitations of Laboratory Measurements

Flow visualizations impossible in non-transparent fluids or non accessible cells



Takeshita et al. (1995)

Global heat transfer (input heating power) and local temperature measurements (thermocouples or bolometers) are the only direct measurements

Too many probes would interfere with the flow

Most of flow features conjectured by indirect evidence!

The numerical simulations

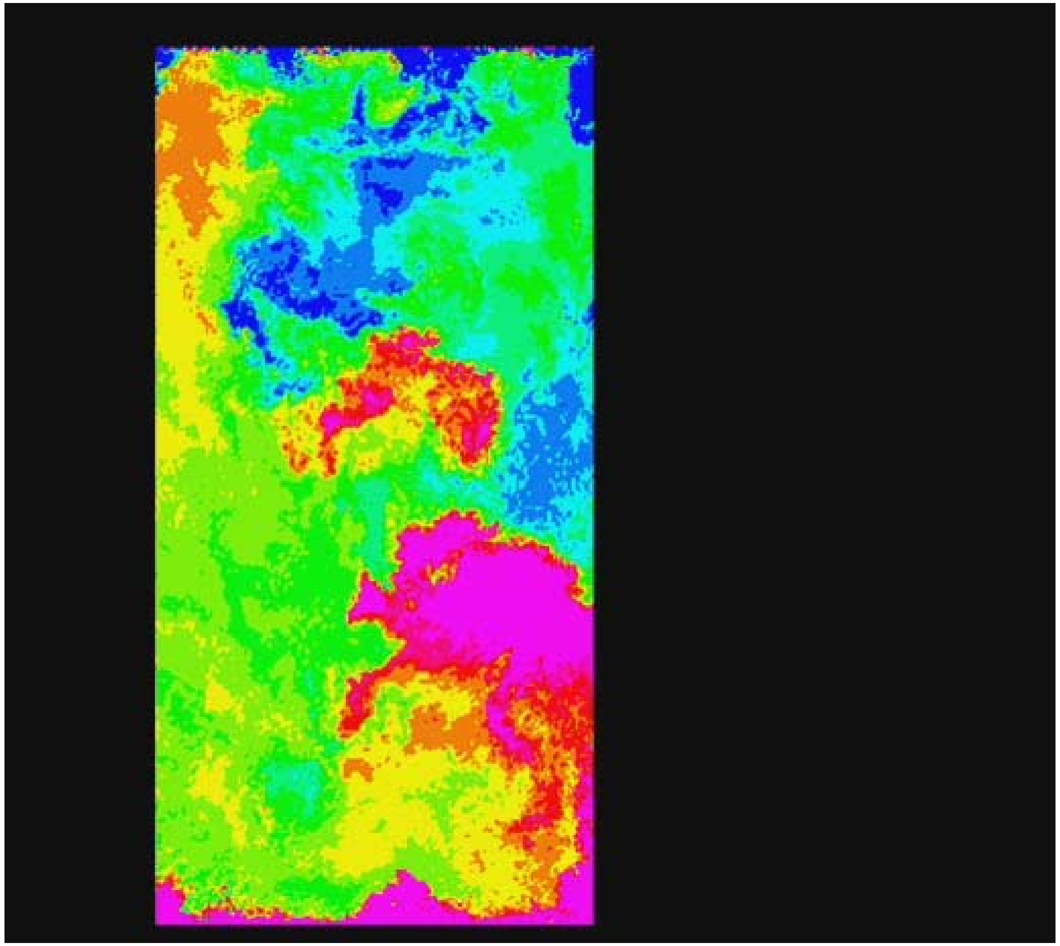
- Pros ☺
  - Flow visualization/how many probes you like!
  - Continuos variations of parameters (***Re, Pr***)
  - Unconditional validity of the approximations (e.g. Boussinesq approx.)
  - Precise assignment of boundary conditions (especially temperature)
- Cons ☹
  - Enough spatial resolution to solve:
    - Thermal and viscous boundary layers
    - Bulk smallest scales
    - Using (really) stretched grid
  - Enough temporal resolution to simulate
    - The fastest flow scales
    - Long time integration to accumulate enough statistics

Numerical Simulations

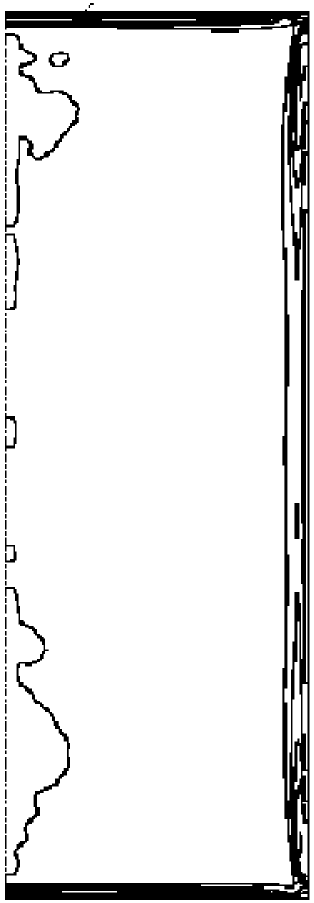
Flow visualizations always possible

Direct measurement of virtually any quantity (real or derived)

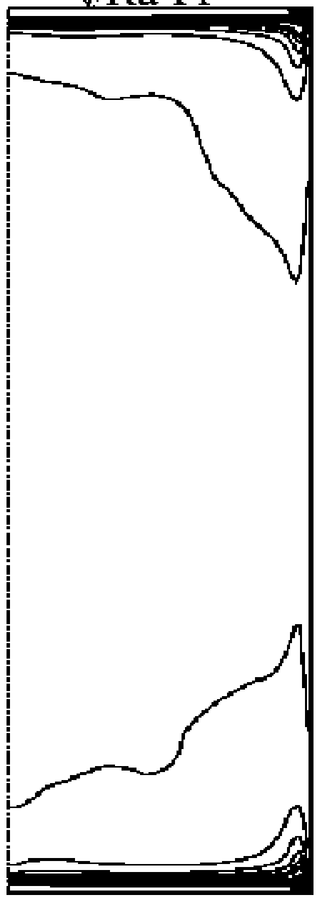
Ra=2.e+13   Pr=0.7



$$\varepsilon = \sqrt{\frac{Ra}{Pr}} (\nabla u)^2$$



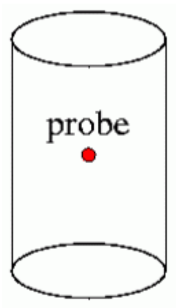
$$N = \frac{1}{\sqrt{Ra\ Pr}} (\nabla \theta)^2$$



Numerical Simulations

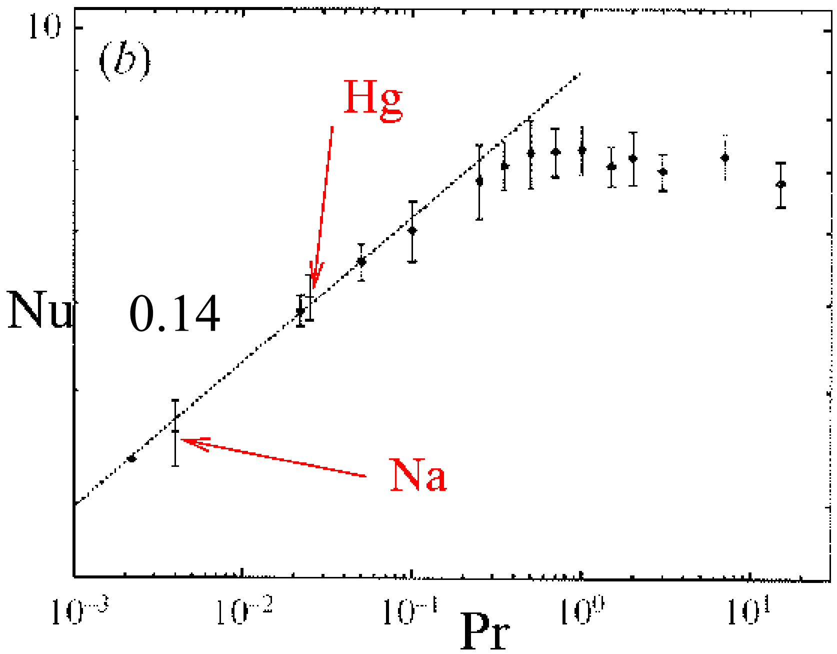
Continuous variation of flow parameters (Ra, Pr) $Pr = \frac{\nu}{k}$

Unconditional validity of the Boussinesq approximation

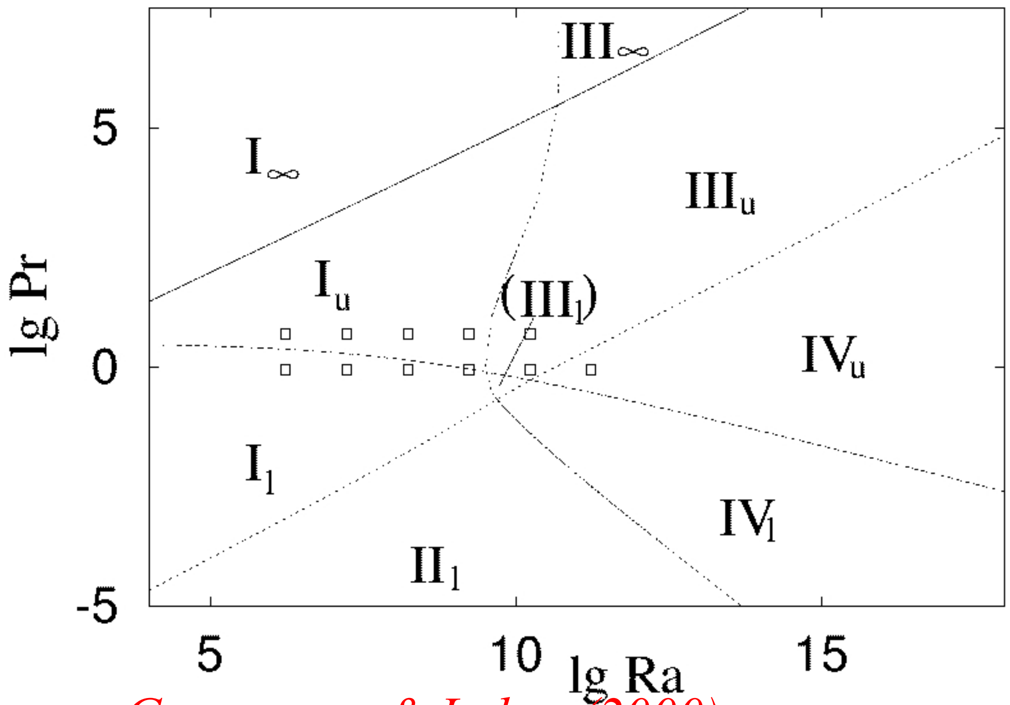


Ideal non-intrusive (numerical) probes

(about 400 probes in the simulations)



Verzicco & Camussi (1999)



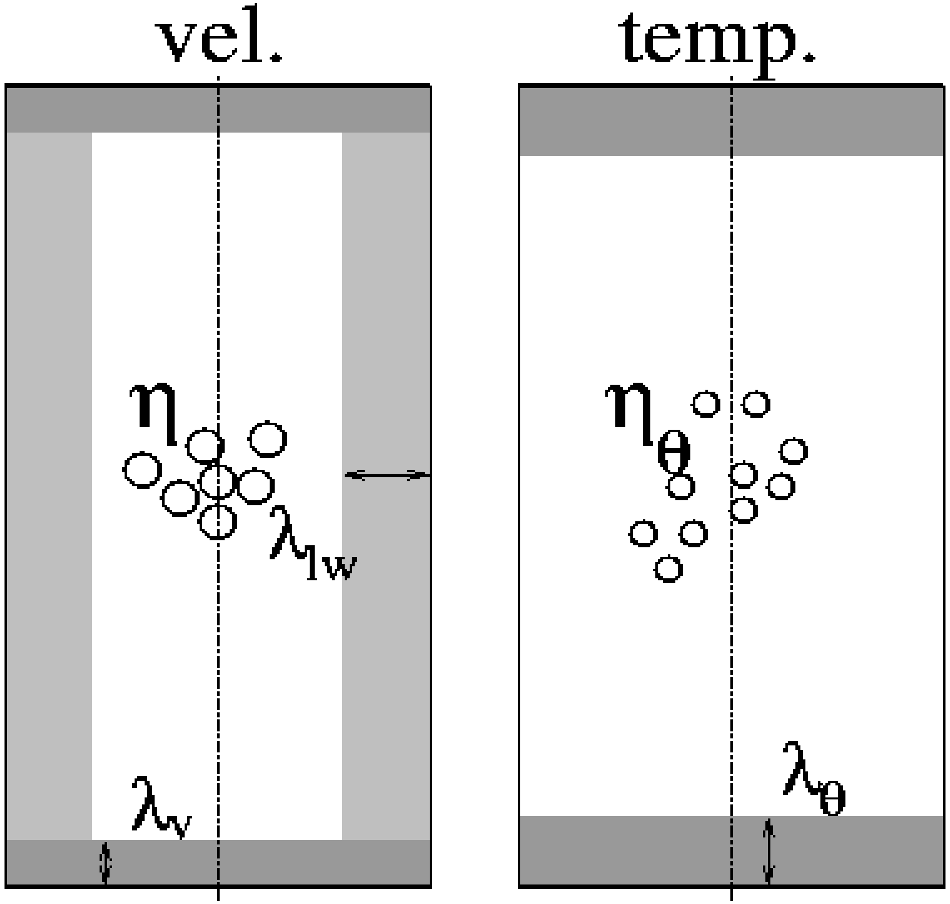
Grossmann & Lohse (2000)

....However (numerical simulations)

... no free lunches ....

In any honest direct numerical simulation all the dynamically relevant flow scales (boundary layers and bulk) MUST be properly resolved

Temp. Pr=0.7 Ra=2.e+11



Resolution Requirements (bulk)

The grid size δ must be of the order of the smallest between Kolmogorov and Batchelor (or Corrsin) scales in the bulk.

$$(Nu-1)Ra = \frac{h^4}{k^2\nu}\epsilon$$ (exact from equations) $$(Nu-1) \approx Nu$$

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$ Kolmogorov scale $$\eta_\theta = \frac{\eta}{Pr}$$ Batchelor scale

$$\frac{\delta}{h} = \mathcal{O}\left(\frac{\eta}{h}\right) = \pi\left(\frac{Pr^2}{RaNu}\right)^{1/4} \qquad Pr \le 1$$

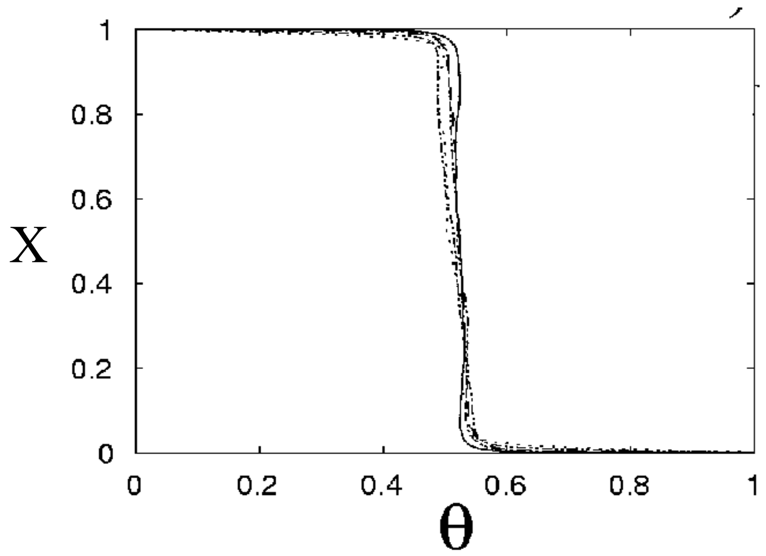
$$\frac{\delta}{h} = \mathcal{O}\left(\frac{\eta_\theta}{h}\right) = \pi\left(\frac{1}{Pr^2 RaNu}\right)^{1/4} \qquad Pr \ge 1$$

Bulk

Grotzbach (1983)

Resolution Requirements (boundary layers)

The thinnest of viscous and thermal boundary layers must contain at least 5-8 grid nodes



Grotzbach (1983) suggested 3. Too few!

$$\frac{\lambda_\theta}{h} = \frac{1}{2Nu}$$ thermal boundary layer

$$\frac{\lambda_U}{h} \simeq \frac{1}{4\sqrt{RaPr}}$$ laminar viscous boundary layer (Blasius type)

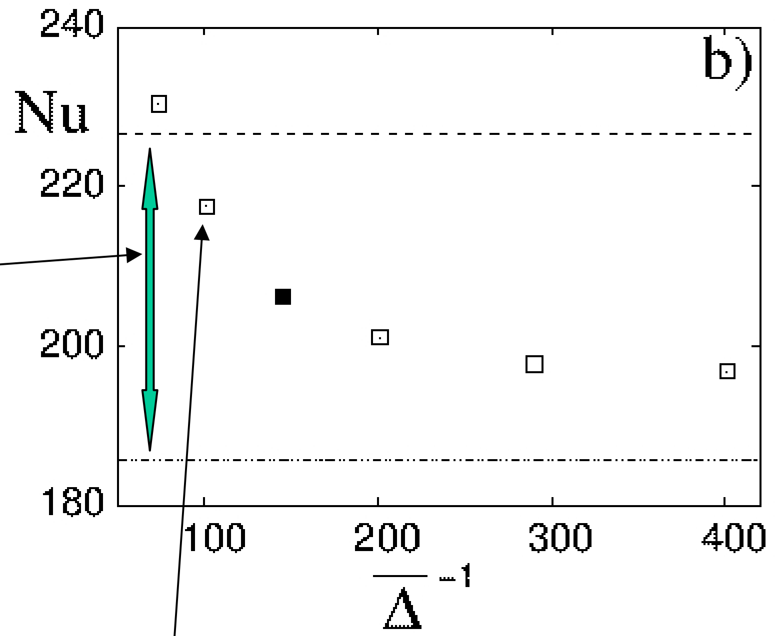
$$\left.\frac{\delta_w}{h}\right|_{wall} \leq \frac{\lambda_\theta}{8h} = \frac{1}{8 \times 2Nu}$$

For moderate and high Pr thermal b.l. is thinner than viscous b.l.

Grid refinement check

The Grotzbach criteria are too mild but a good guideline

Pr=0.7 Ra=2.e+10



Error bar

Grotzbach (1983) criterion

Verzicco &Sreenivasan (2008)

Resolution Requirements (time)

Grotzbach (1983) suggested a fixed number of time steps (200).

The time step size must be of the order of the Kolmogorov time (*Ra* dependent)

The time integration of the equations must be stable (the limit is scheme-dependent)

(3rd order R-K)

$$\frac{\Delta t}{T} \leq \frac{t_\eta}{T} \simeq \frac{1}{\sqrt{RaPr}}$$

$$\frac{\Delta t U}{\delta} = CFL \leq \sqrt{3}$$

Numerical stability is usually more restrictive than physical limit

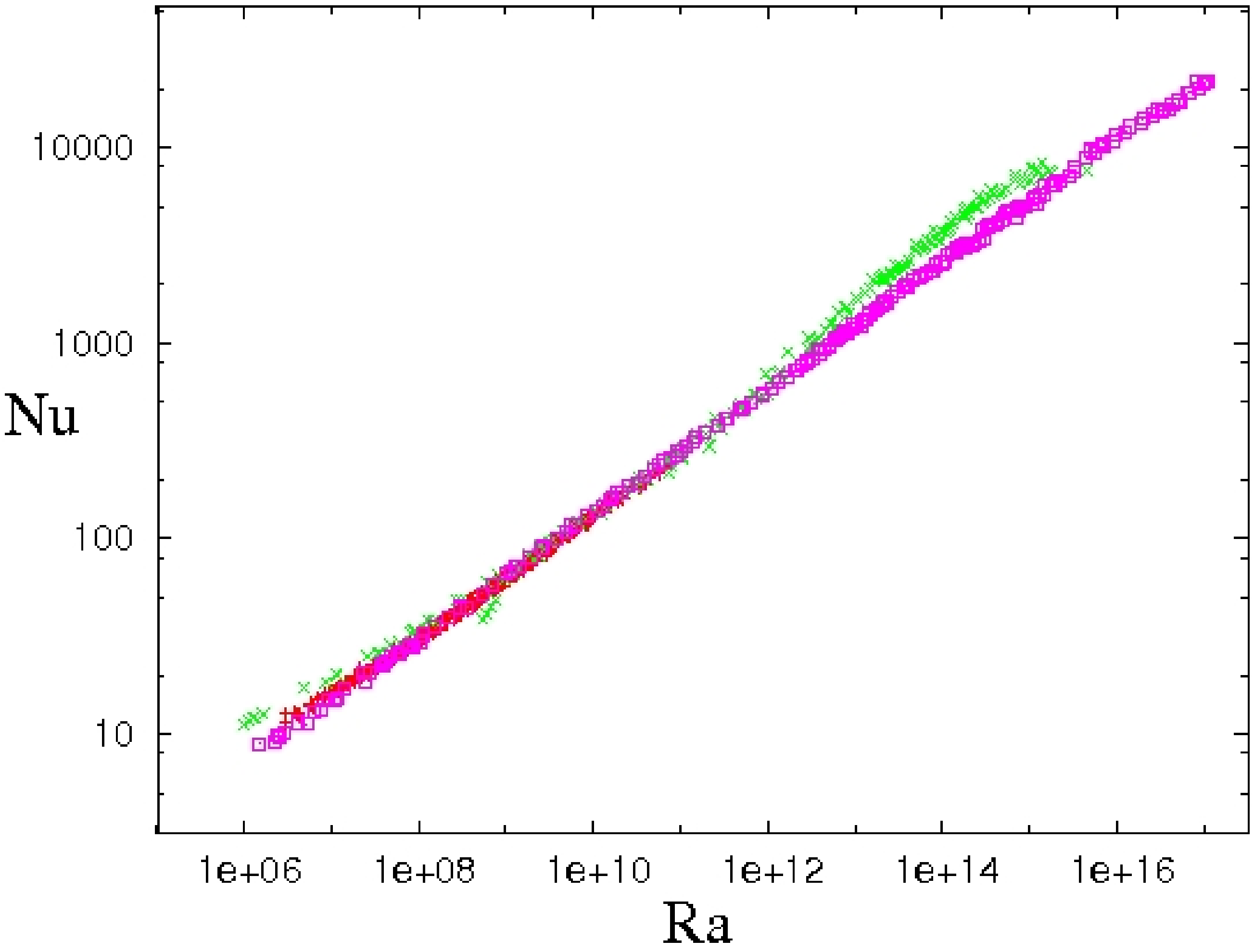
800 time steps each large-eddy-turnover-time at Pr=0.7 and Ra=2.e+11

2000 time steps at Ra=2.e+12 (2.6e+04 CPU hours for 100 T)

5000 time steps at Ra=2.e+13 (1.5e+05 CPU hours for 50 T)

State-of-the-Art Experiments (high Ra)

Cryogenic helium, cylindrical cell *Γ=1/2*

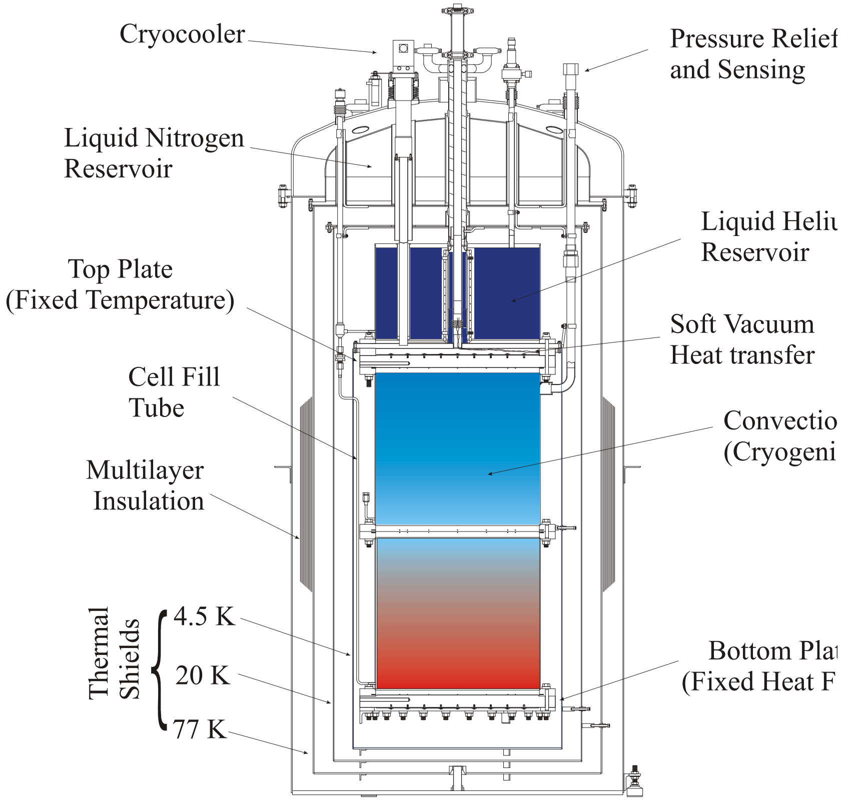


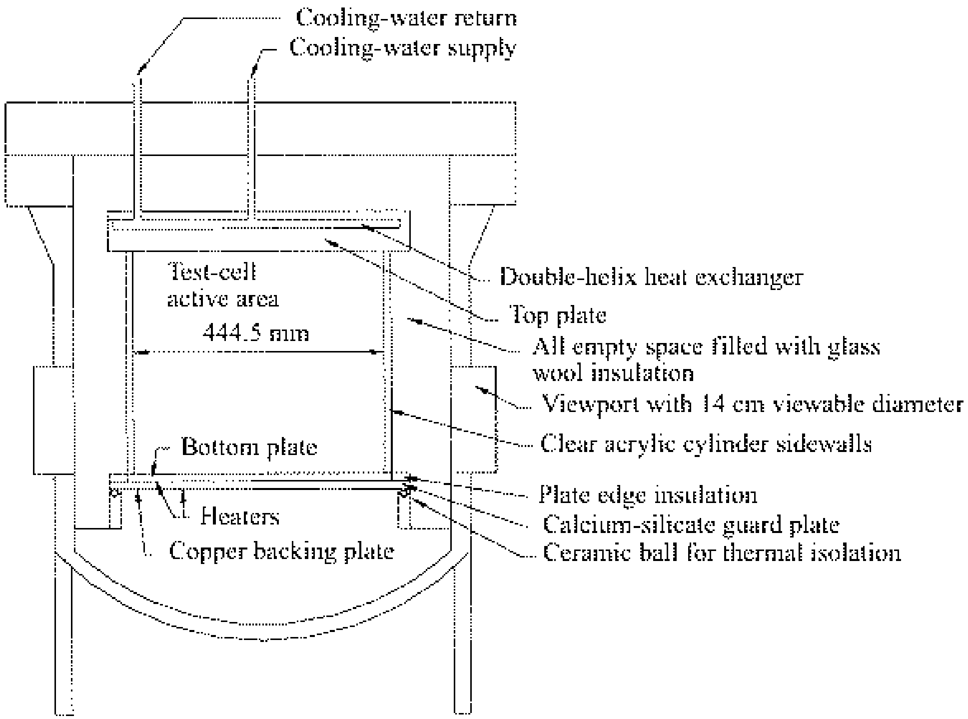
Niemela et al. (2000), Chavanne et al. (2001), Roche et al. (2002)

Why a Low-Aspect-Ratio Cylindrical Cell?(Exp.)

Most of the experimental set-ups rely on large pressure variation to achieve large *Ra* range within the same experimental apparatus

Sidewalls have to withstand with huge pressure forces without deforming and the cylindrical geometry is the most practical.





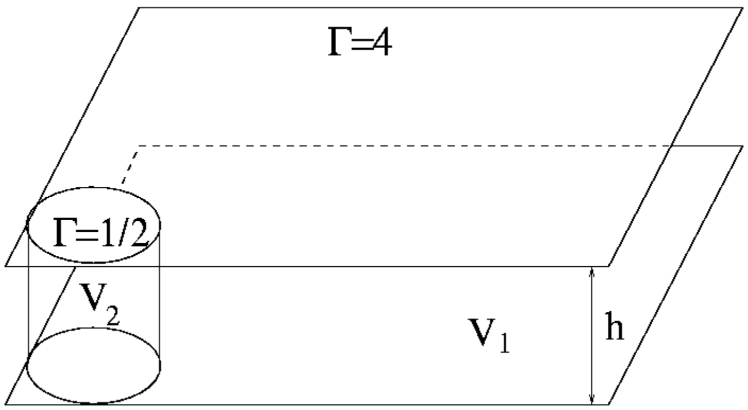
Niemela et al. (2000), Chavanne et al. (2001), Roche et al. (2002)

Fleischer & Goldstein (2002)
Ashkenazi & Steinberg (1999)

Why a Low-Aspect-Ratio Cylindrical Cell? (DNS)

To make close contact with some state-of-the-art experiments

(to date the highest Rayleigh number experiments have been performed in a cylindrical cell of aspect-ratio Γ=1/2 *Niemela et al., 2000, Chavanne et al., 2001, Roche et al., 2002)*



$$V_1 = 16h^3$$

$$V_2 = 0.19h^3$$

At Ra=2.e+14 to maintain in V_1 the same spatial resolution as in V_2 ≈1.e+11 nodes would be needed: presently unfeasible!

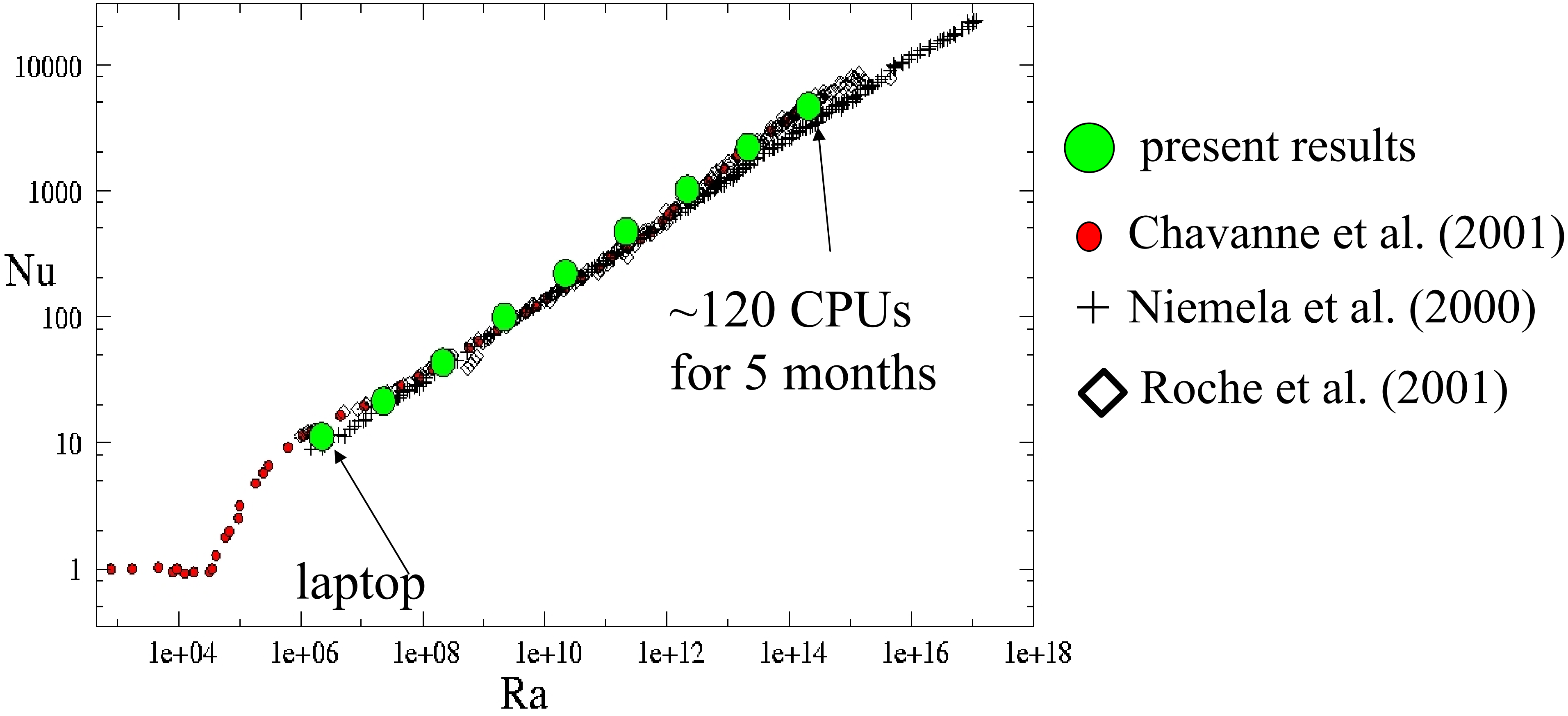
At *Γ=4* → *Ra=2x10⁷* *(Kerr, 1996)*

At *Γ=10* → *Ra=10⁶* *(Shishkina & Wagner, 2006)*

Aspect ratio *Γ* has to be traded with *Ra*

Results (heat transfer)

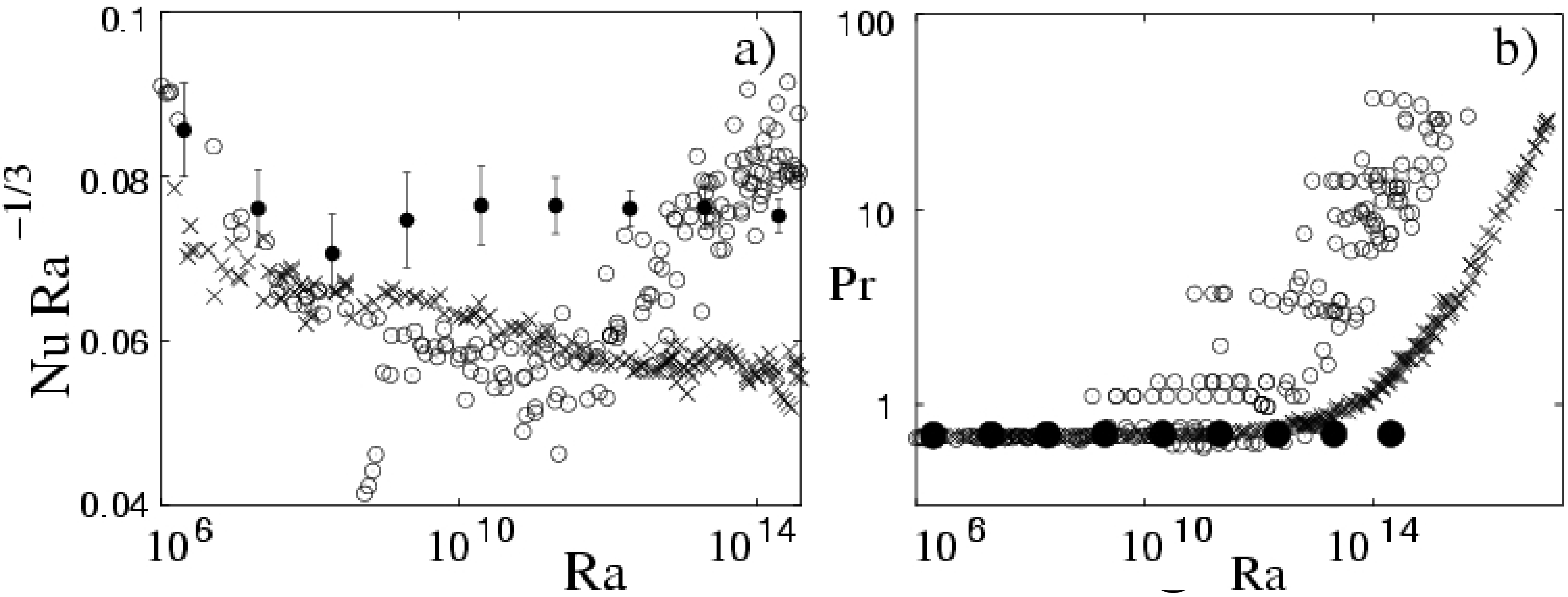
Experiments in a cyl. cell with Γ=0.5 in cryogenic helium



The results "seem" in good agreement with experiments, BUT ……

Results

Amati et al. (2005)



Owing to Pr variation experiments and simulation in different regions of the Ra-Pr plane

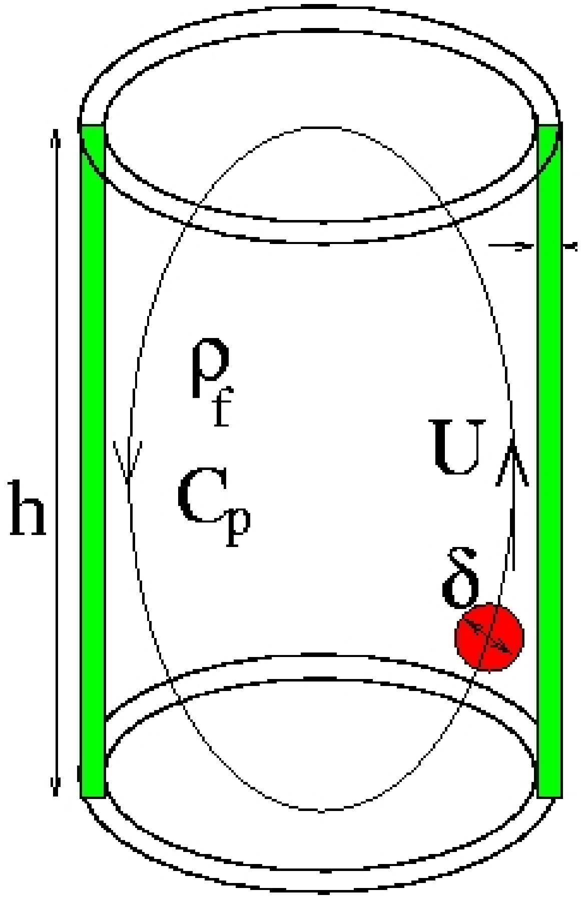
Different mean flow structures *(Stringano & Verzicco, 2005)?*

Mean flow structure

Stringano & Verzicco (2005)

Plume time $t_f = \frac{\delta_\theta^2}{k_f} = \frac{\delta_\theta^2 \rho_f C_p}{\lambda_f}$ $\quad \delta \approx \delta_\theta = \frac{1}{2Nu}$

Convective time $t_U = \frac{h}{U} = \frac{h}{\sqrt{g\alpha\Delta h}}$ $\quad Nu = ARa^\beta$



For the recirculation to exist it must be $\frac{t_U}{t_f} << 1$

$$\frac{t_U}{t_f} = \frac{1}{\sqrt{RaPr}} \left(\frac{h}{\delta_\theta}\right)^2 = \frac{4A^2 Ra^{2\beta}}{(RaPr)^{1/2}}$$

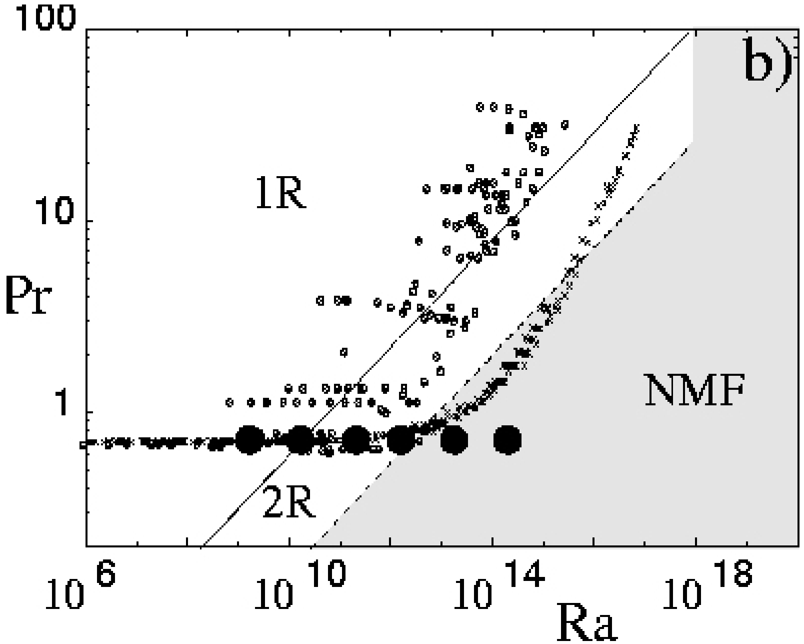
Since β>1/4 the plume dimension sets the limit $\frac{t_U}{t_f} = 1$

If the plume is too thin it can not travel the distance h since it diffuses; it can however travel a shorter distance and then sink again

➔ cell break-up.

Results

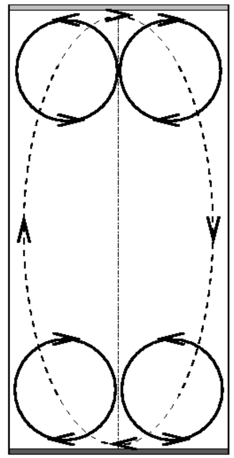
adapted from Stringano & Verzicco (2005)



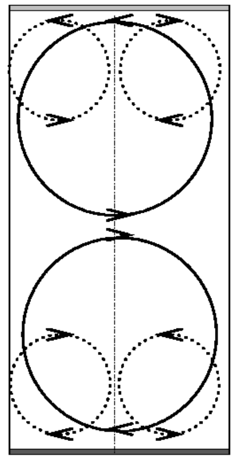
X Niemela et al. (2000)

○ Chavanne et al. (2001)

● present results



1R

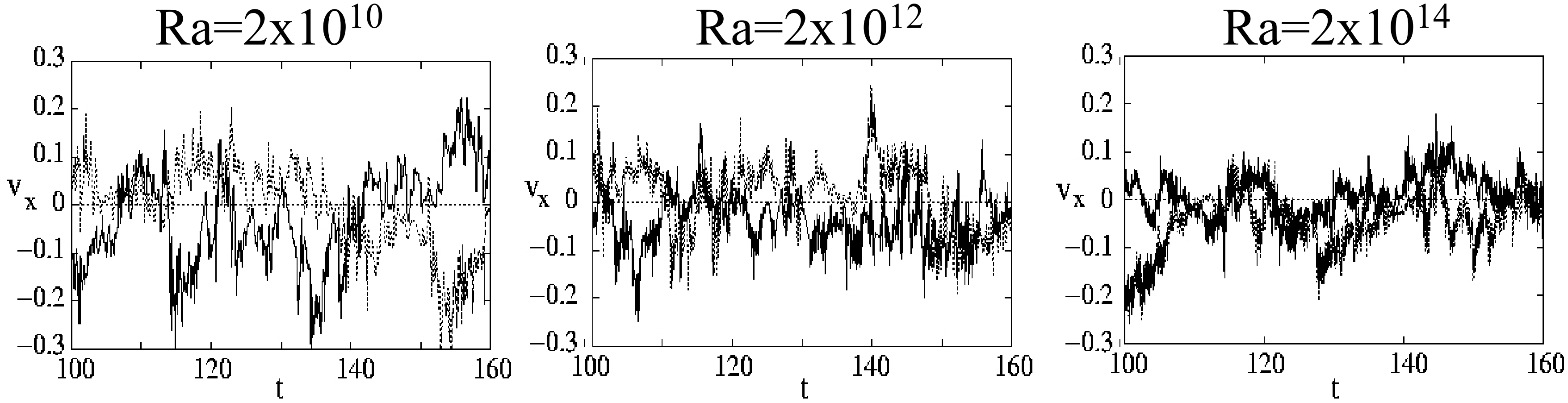


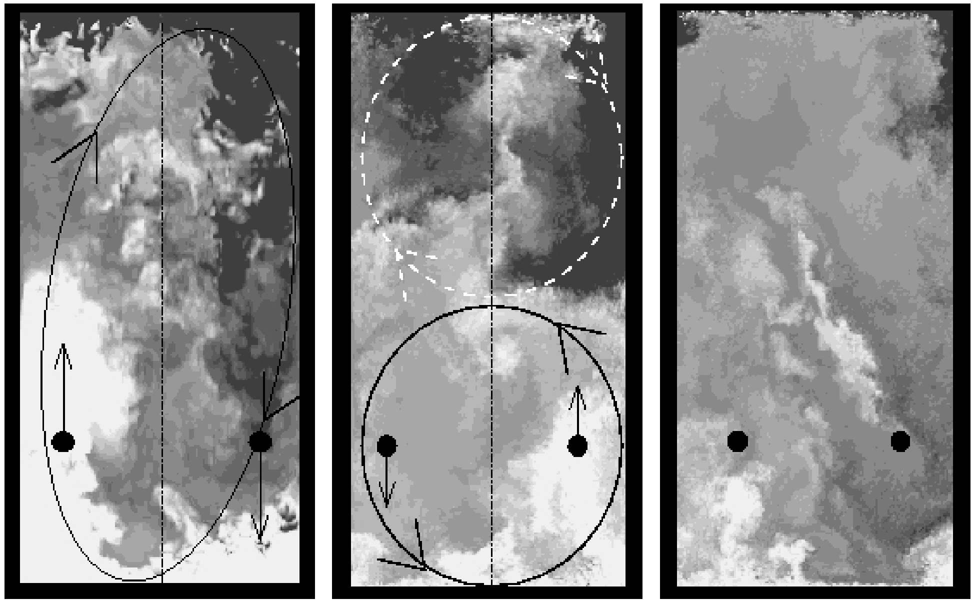
2R

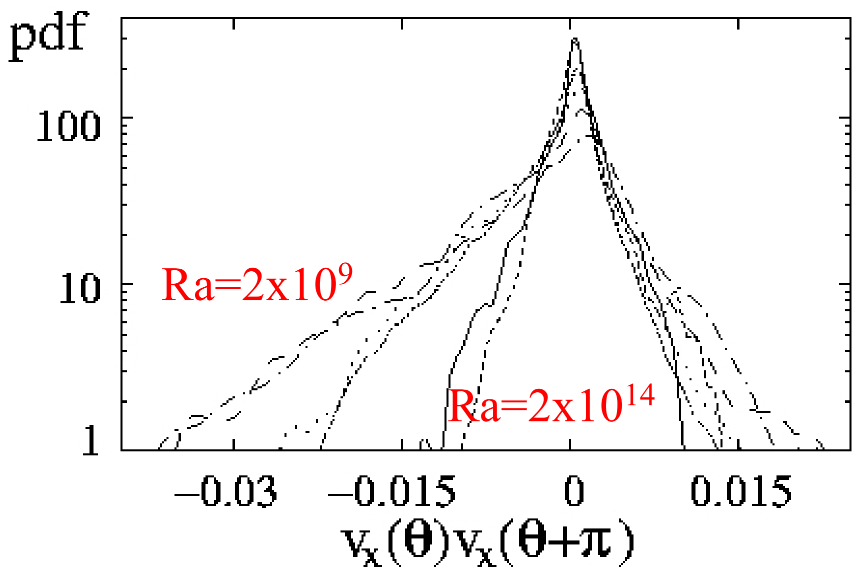
Only the numerical simulations really enter the no-mean-flow region (NMF)

The absence of mean flow (NMF region) implies "disconnected" thermal b.l. and **Nu ~ Ra$^{1/3}$** Malkus (1954).

Velocity statistics

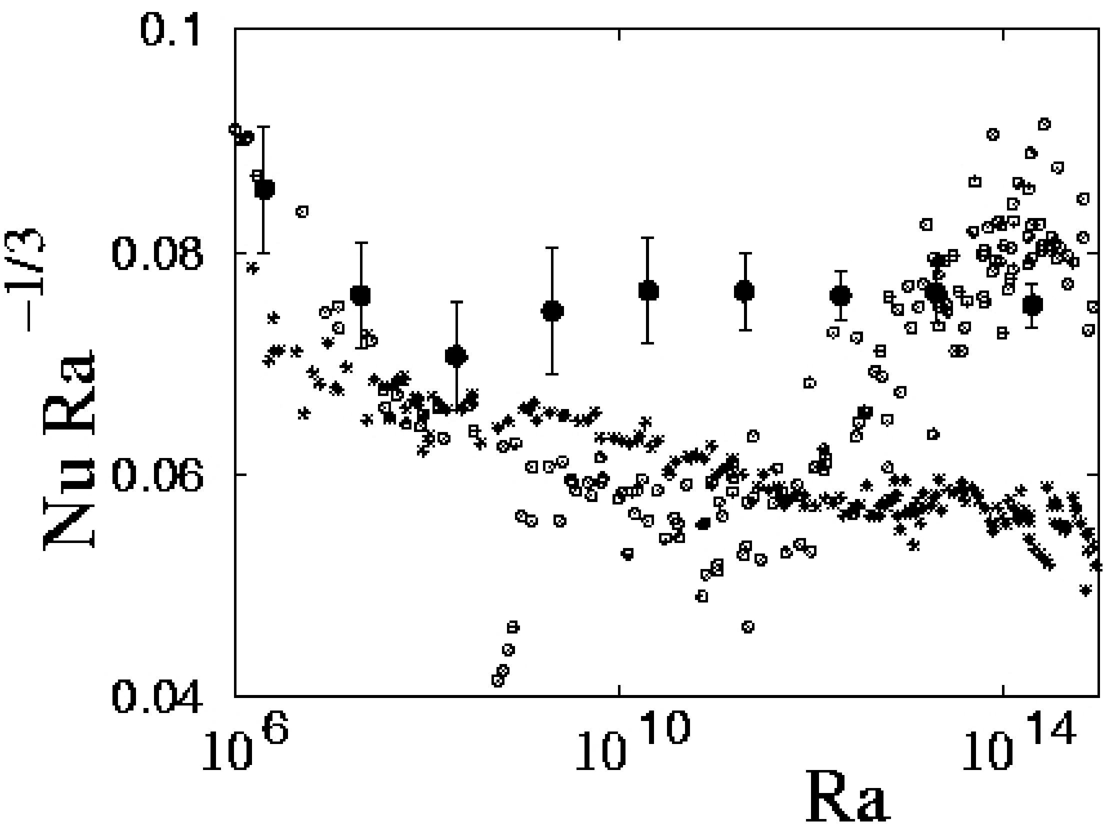






Indeed the symmetric PDF implies the absence of a persistent mean flow.

Heat transfer mismatch



Different temperature boundary conditions?

In the numerical simulations the temperature is strictly constant on the plates.

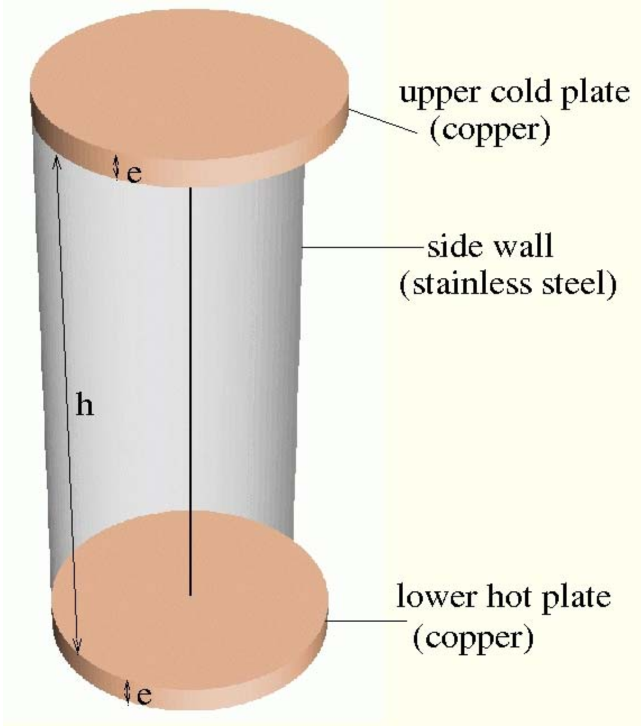
Real plates do not have infinite heat capacity and might have different temperature b.c.

A Rayleigh-Bénard cell

Working fluid: *water, air, liquid metals (mercury, sodium), pressurized gas, silicon oils, cryogenic pressurized gaseous helium.*

Side wall: *stainless steel, plexiglas (high mechanical properties, poor heat conduction)*

Plates: *copper, brass, aluminium sapphire, oxygen free pure copper (high mechanical properties, very good heat conduction)*



The arrangement is such to minimize the heat leakage through the sidewall.
There are corrections for the sidewall (important only at small **Ra**)
Ahlers (2001), Roche et al. (2001), Verzicco (2002), Niemela & Sreenivasan (2003)

The finite conductivity of the horizontal plates alters the heat transfer.
There are reliable corrections (important at high **Ra**)
Chaumat et al. (2002) Verzicco (2004), Brown et al. (2005)

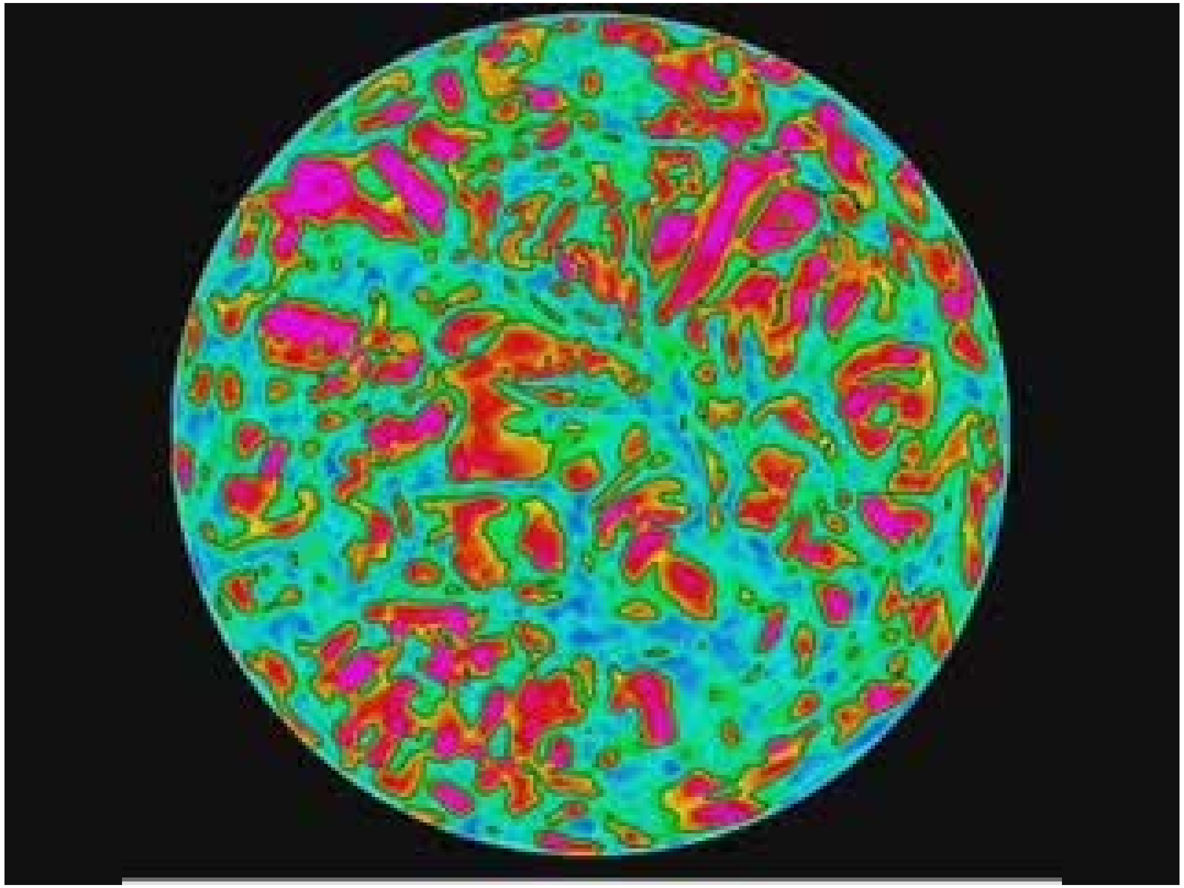
Wall temperature gradient

θ_{wall}=const
Pr=0.7 Ra=$2x10^{10}$
Nu

Convection is strongly unsteady

Mean flow "rotations" and "cessations"

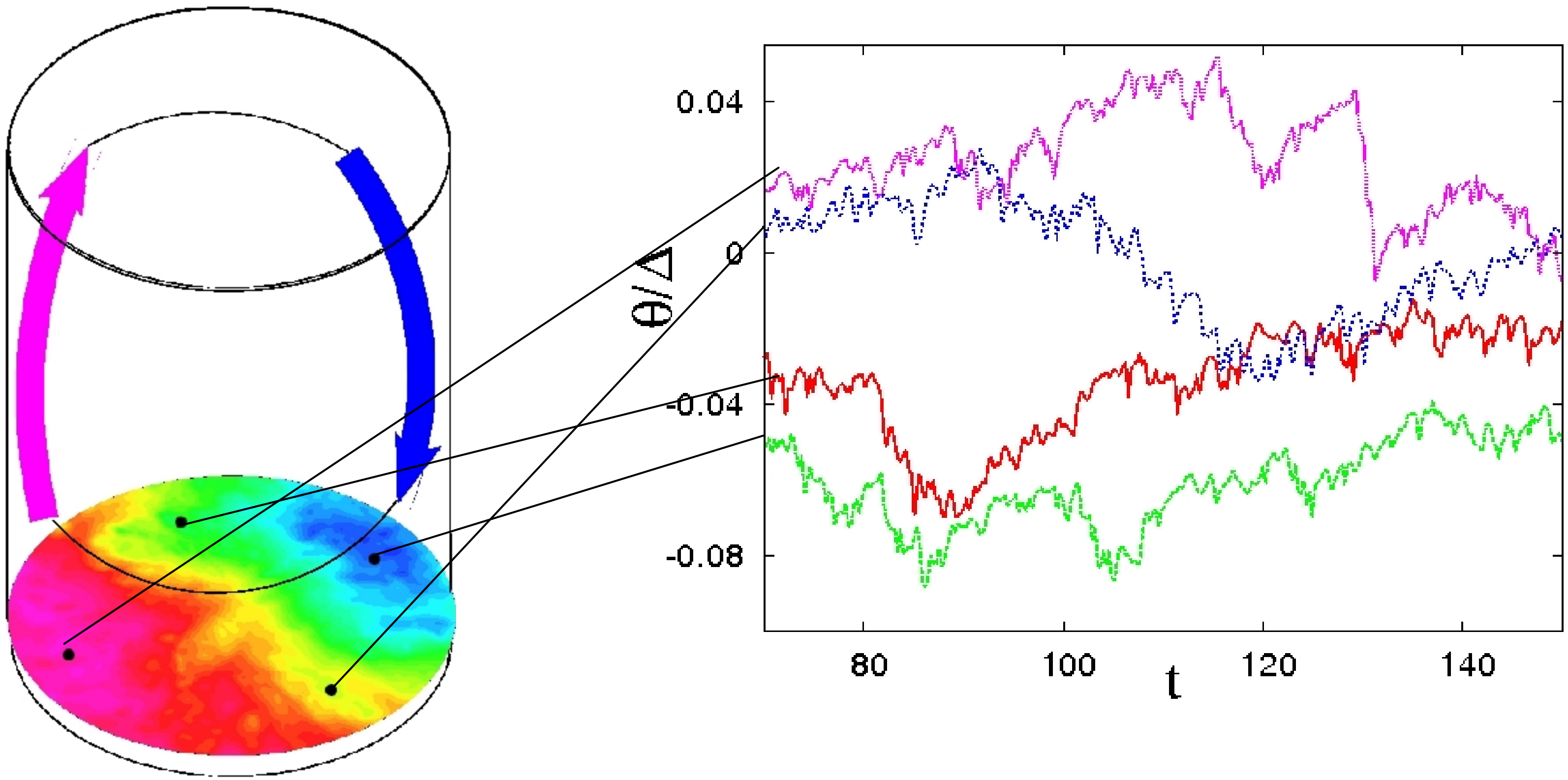
Formation of line plumes



How the plates react to this unsteadiness?

Temperature dynamics in the plate

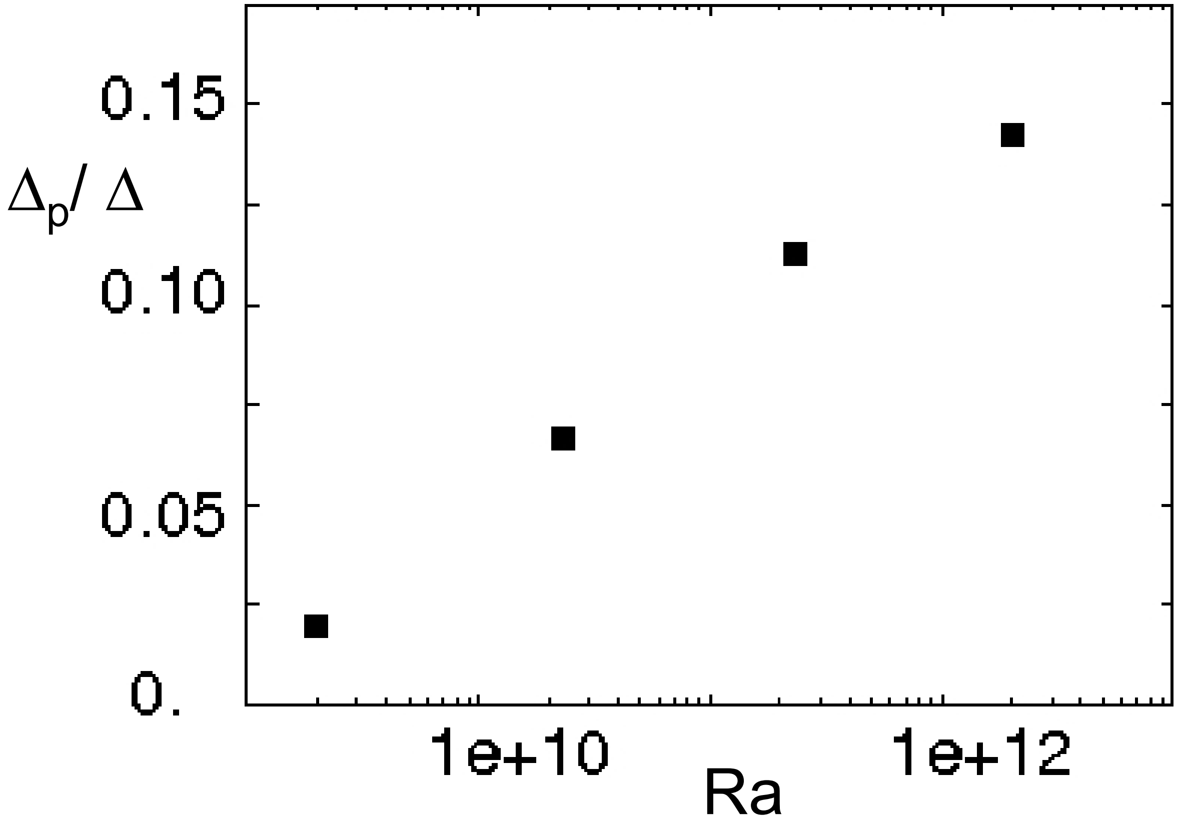
The temperature equation is solved in the solid plate with the heat flux b.c. coming from the flow simulation



Large scale flow footprint on the plate/fluid interface

Temperature dynamics in the plate

The temperature inhomogeneity increases with *Ra*

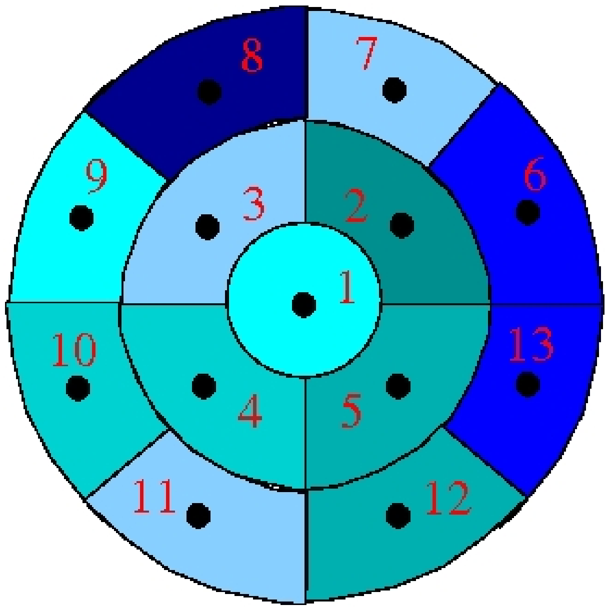


Δ_p / Δ up to 15% for a water/cu combination at $Ra = 2x10^{12}$
and plate thickness $e = 5\%h$

A possible remedy

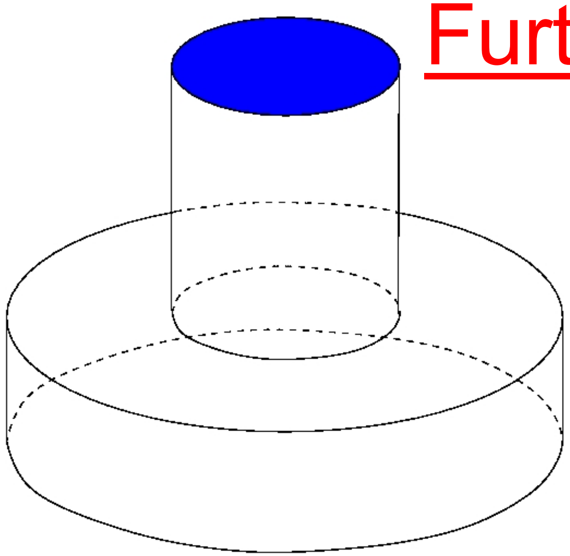
Indeed in many experiments the heating/cooling systems have a feedback loop control to maintain the mean temperature constant.

This, however does not avoid temp. differences on the surface

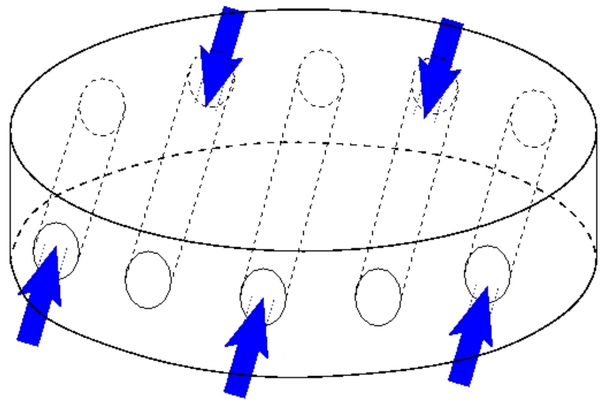


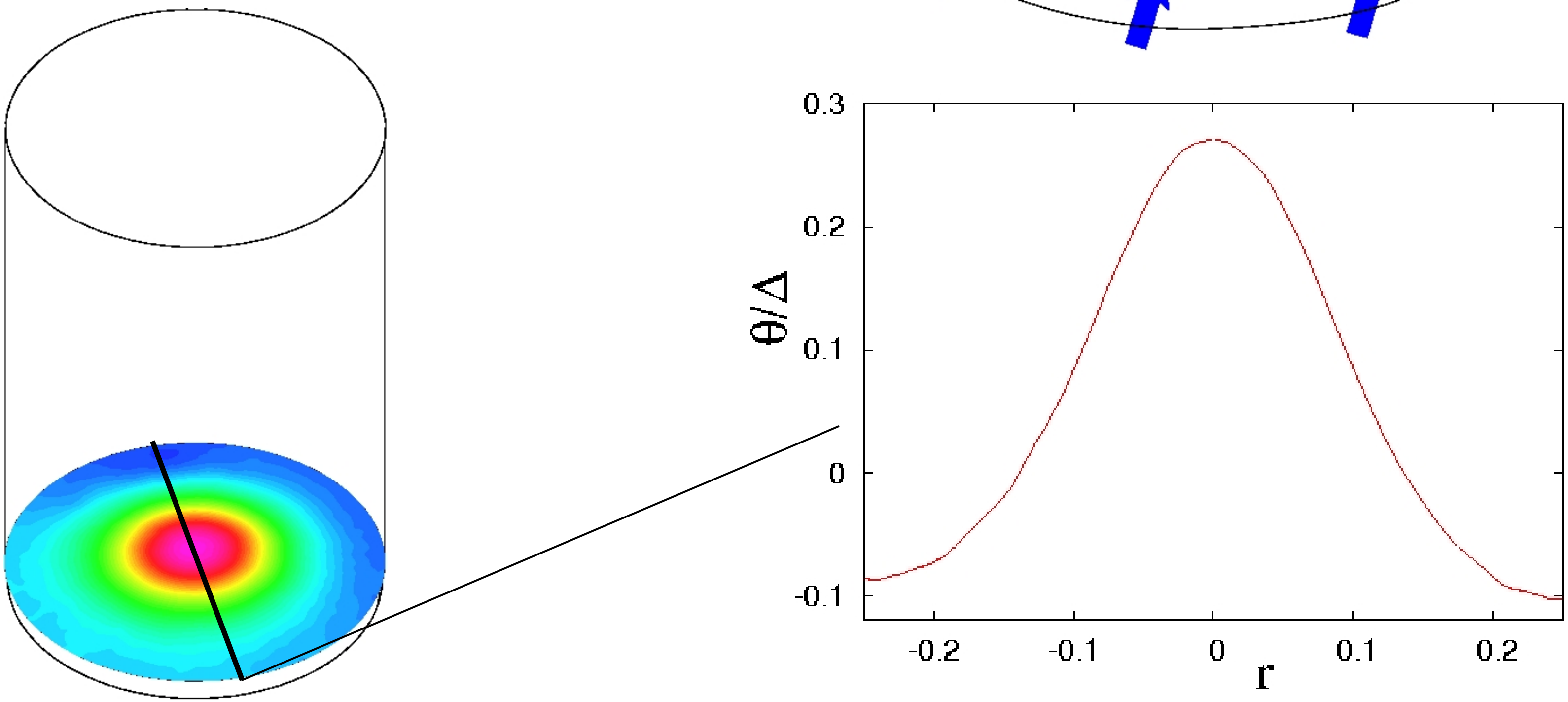
Maybe a plate with several independently controlled sectors would perform better

Further temperature perturbations



Inhomogeneous heat/temperature sources

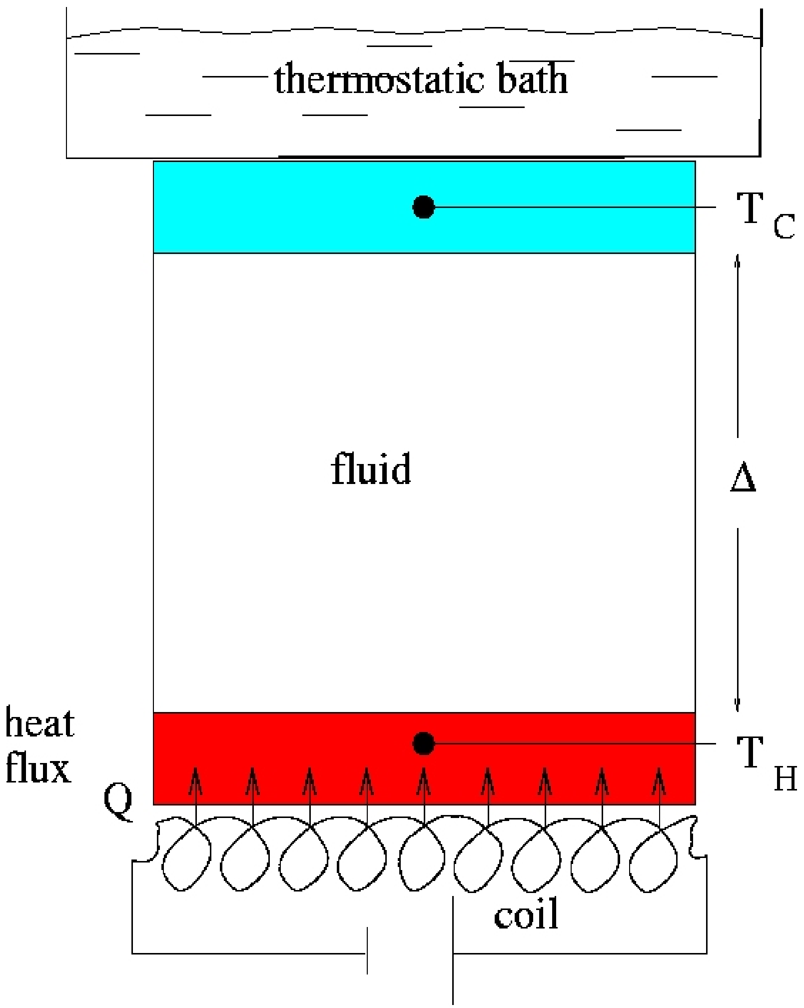




θ/Δ up to 40% for a water/cu combination at $Ra=2x10^{12}$

Plates heating

In most experimental set-ups upper and lower plates are heated and cooled by different methods



The upper plate is in contact with a constant temperature surface and its high thermal conductivity keeps the temperature constant *(the thinner the better)*

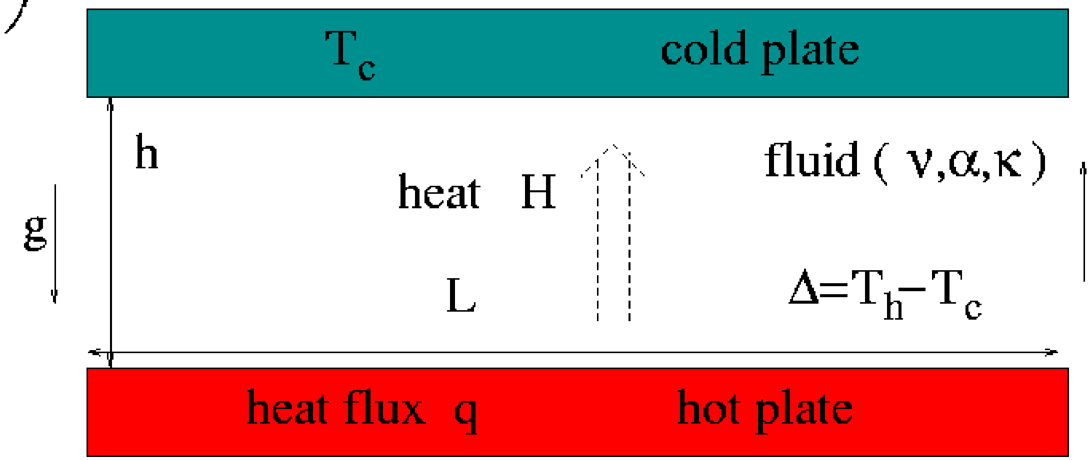
The lower plate has a constant heat flux surface and its heat capacity keeps the temperature constant *(the thicker the better)*

Present problem

Non-dimensional Navier-Stokes equations with the Boussinesq approximation

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta \hat{z} + \left(\frac{Pr}{Ra_q}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{D\theta}{Dt} = \frac{1}{(Pr Ra_q)^{\frac{1}{2}}} \nabla^2 \theta$$



$$Ra = \frac{g\alpha q^4 h}{\upsilon\kappa}$$ `forcing' parameter

$$\left(Ra = \frac{Ra_q}{Nu} \right)$$

On input: $\mathrm{Pr} = \frac{\upsilon}{\kappa}$ *fluid properties* On output: T_h

$$\Gamma = \frac{d}{L}$$ *geometric parameter*

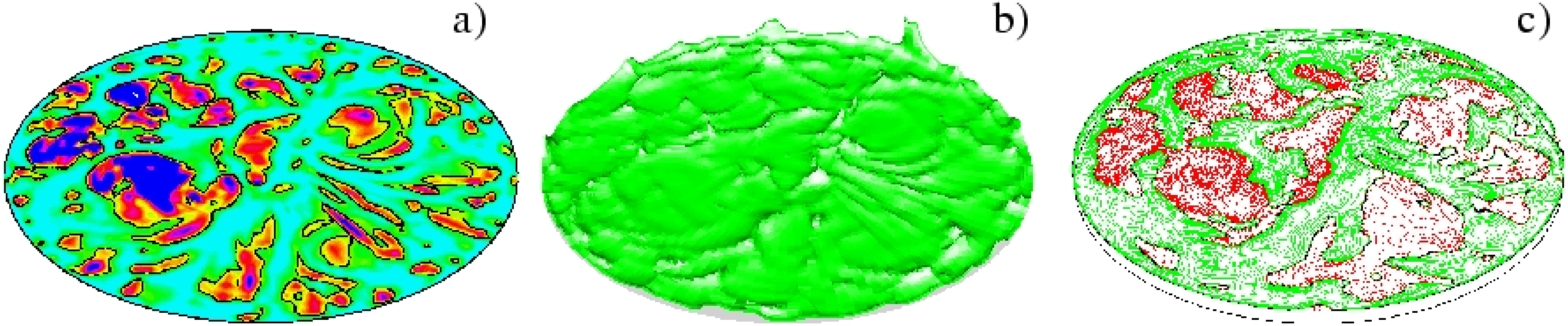
$$\left(Nu = \frac{qh}{\Delta} \right)$$

Constant temperature dynamics

Near wall dynamics *(θ =const)*

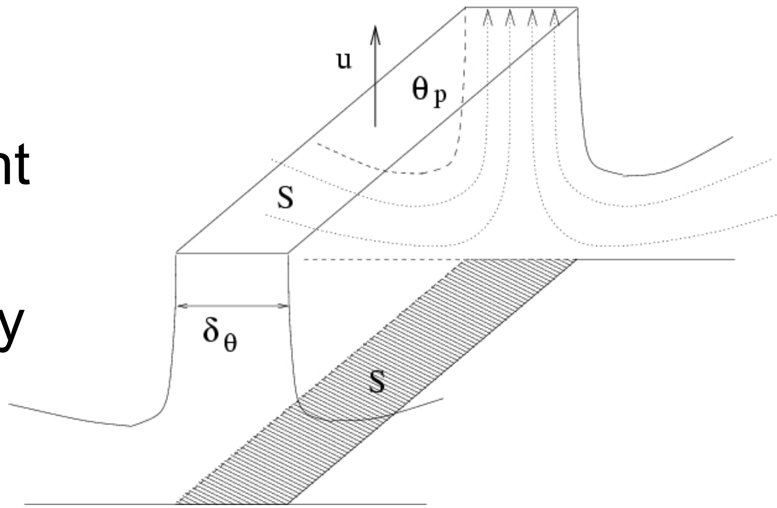
$Pr=0.7 \quad Ra=2x10^9$

$(\partial\theta / \partial z)_{wall}$ $\theta=0.8 <\theta_{wall}>$ u_z



The plate is swept on the sides of a plume

The wall temperature gradient increases above the average



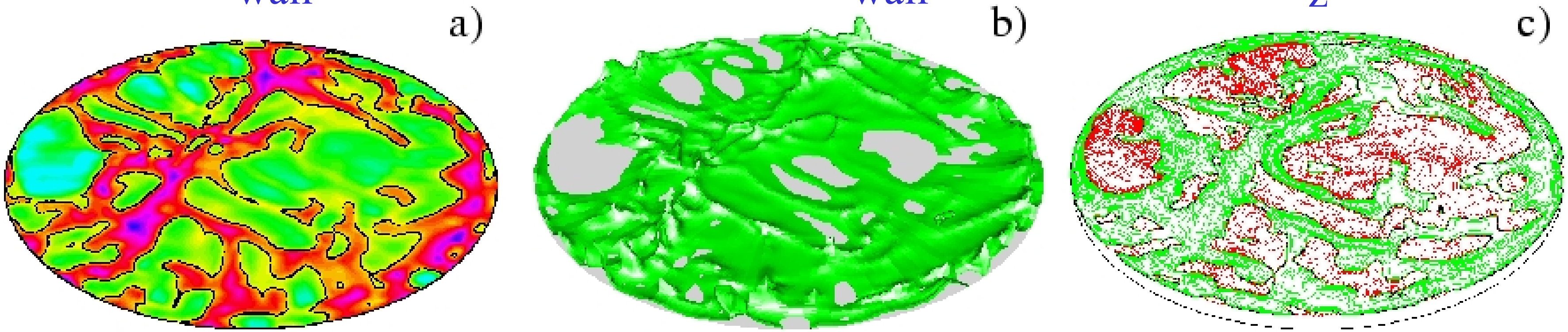
The fluid below the plume is stagnant

The flow can provide any heat flux by making the thermal b.l. thinner

Constant heat flux dynamics

Near wall dynamics *(q=const)* $Pr=0.7 \quad Ra=2\times10^9$

θ_{wall} $\theta=0.8 <\theta_{wall}>$ u_z

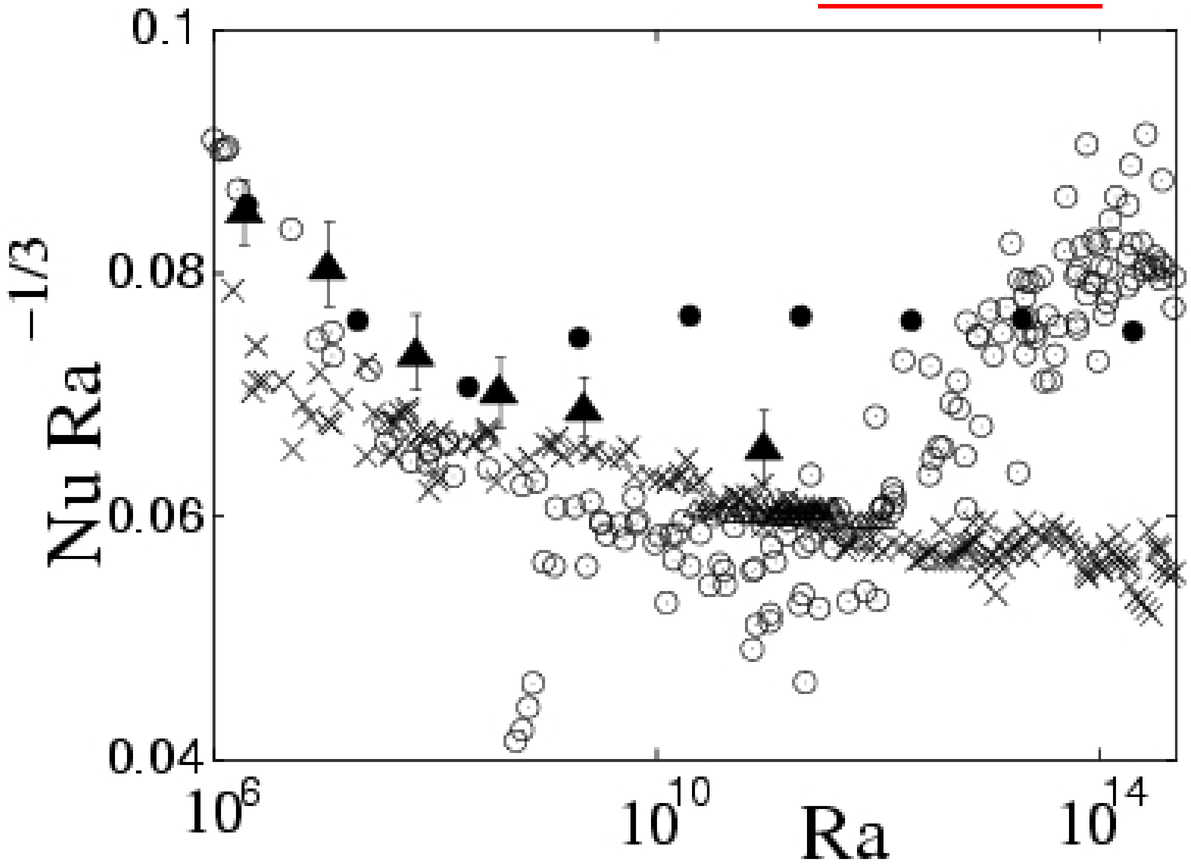


The plate cools down during the formation of a plume

The wall temperature decreases below the average

The resulting plumes are colder and carry less heat

Results



- • Amati et al. 2005
- o Chavanne et al. 2001
- x Niemela et al. 2000
- Δ Nikolaenko et al. 2005
- ▲ Verzicco & Sreenivasan (2008)

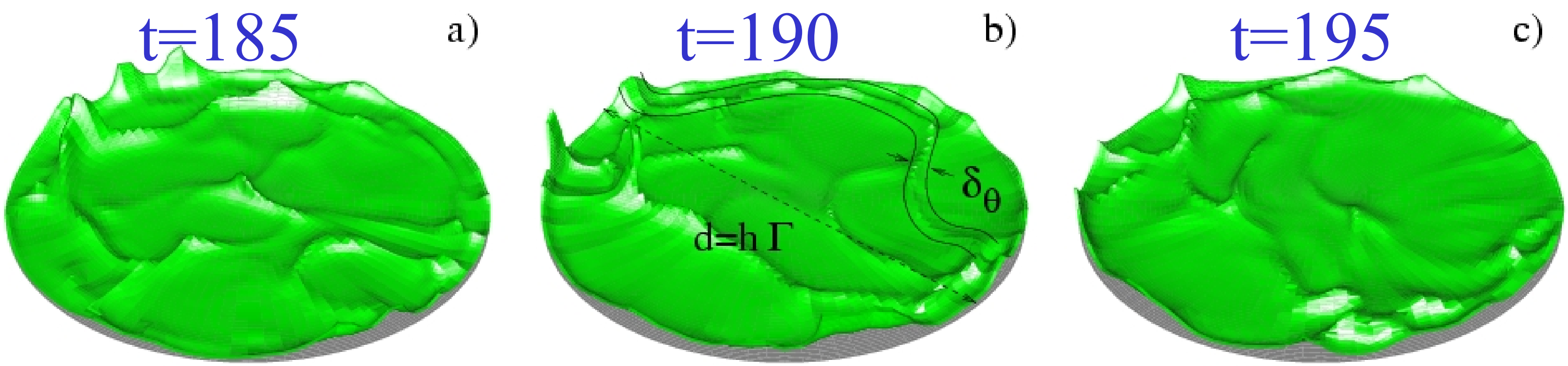
For Ra≥10^9 simulations closer to experiments.

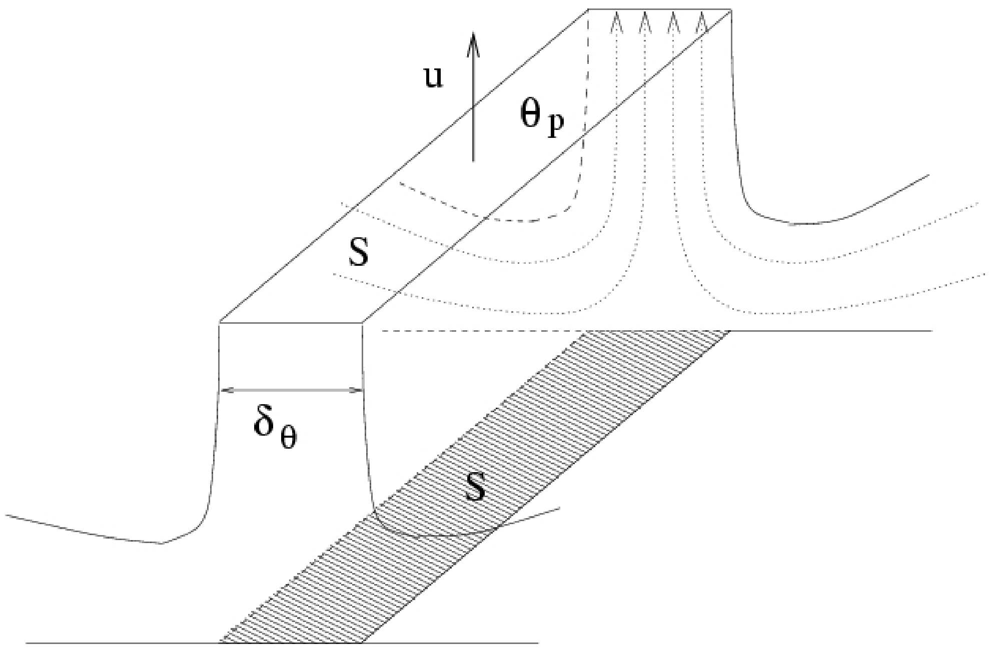
Note: unlike the simulations, experiments have a plate between the heater *(q=const.)* and the fluid.

Classical "puzzle" still unsolved.

A simple model

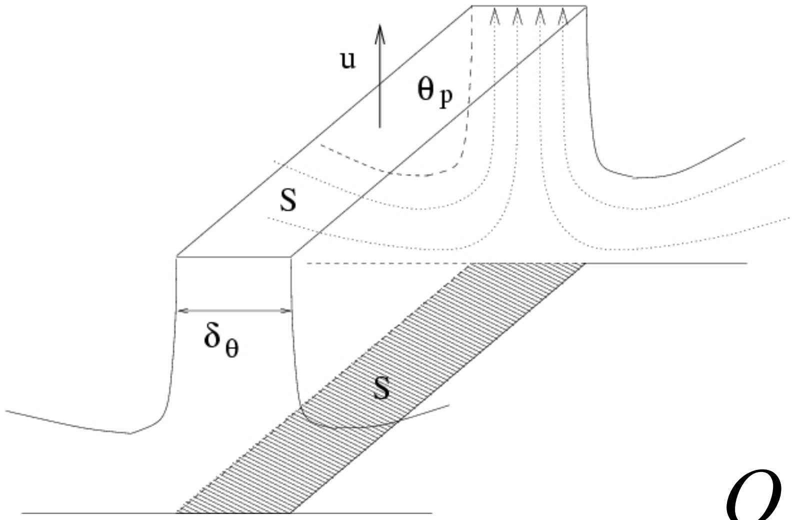
θ_{wall}=const
Pr=0.7 Ra=2x10^8





(line) plumes have the same thickness as the thermal boundary layer and a horizontal extension comparable with the cell size

A simple model



Heat flux needed by a plume

$$Q_p \approx \rho C_p \vartheta_p u S$$

Average heat flux through a surface element S

$$Q_w \approx \lambda < \partial\theta / \partial z >_w S = Nu \lambda \Delta / h$$

If: $\theta_p \approx \Delta$ (a plume is a piece of detached b.l.)

$u \approx g\alpha\Delta\delta_\theta h\Gamma / \upsilon$ (buoyancy and drag in equilibrium)

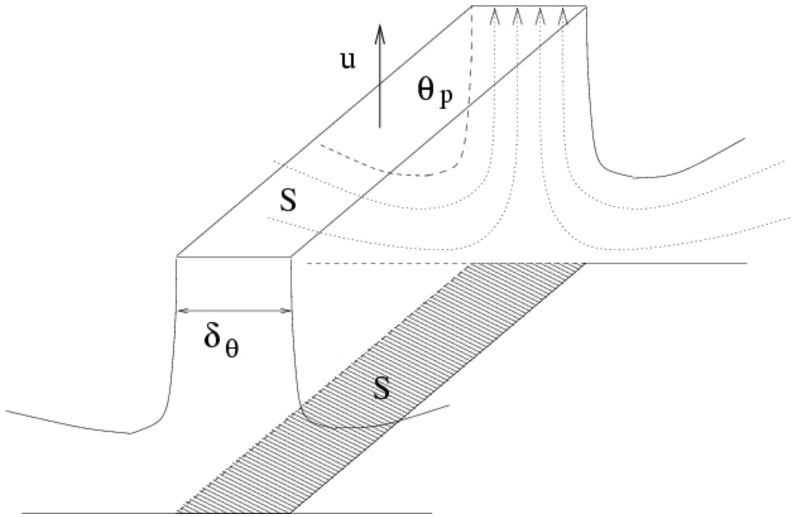
(Castaing et al. 1989)

$$\frac{Q_p}{Q_w} \approx \frac{Ra}{Nu^2}$$

which increases with Ra if $Nu \sim Ra^\beta$ with $\beta < 1/2$

Note $\theta_p \approx \Delta$ $\forall Ra$ only if $\theta_{wall} = const$

A simple model



If: $< \partial\theta / \partial z >_w = const$

a plume can not drain more heat than that provided by the wall

$$\frac{Q_p}{Q_w} \approx 1$$

The plume temperature θ_p can be computed

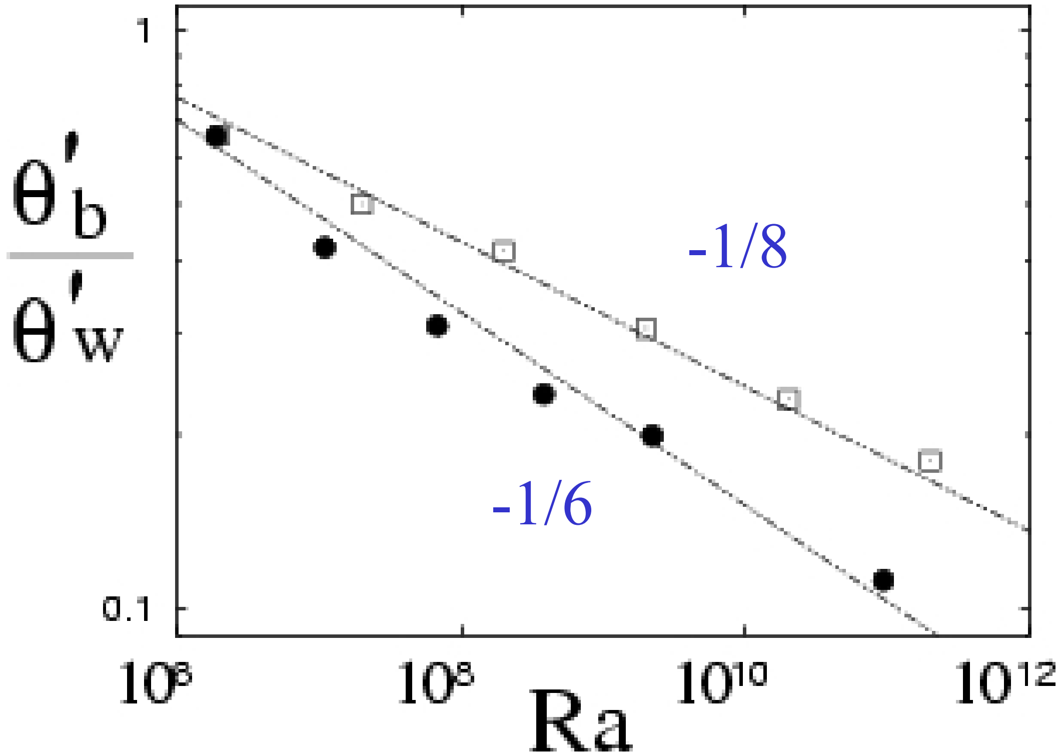
$$\theta_p \approx \Delta \frac{Nu}{Ra^{1/2}}$$

which decreases with Ra if $Nu \sim Ra^{\beta}$ with $\beta < 1/2$

Note similar conclusions if $u \approx \sqrt{g\alpha\Delta h}$ (free fall velocity)

or $u \approx \sqrt[3]{g\alpha < u_z^{'}\theta^{'} > h}$ *(Hunt et al. 2003)*

A simple model



$\frac{\theta_b'}{\theta_w'}$ is the fraction of wall temperature fluctuations that reach the bulk: PLUMES?

● *q=const*

□ *θ=const*

$$\theta_p \approx \Delta \frac{Nu}{Ra^{1/2}}$$ with $Nu \sim Ra^{1/3}$ yields $$\theta_p \approx \Delta Ra^{-1/6}$$

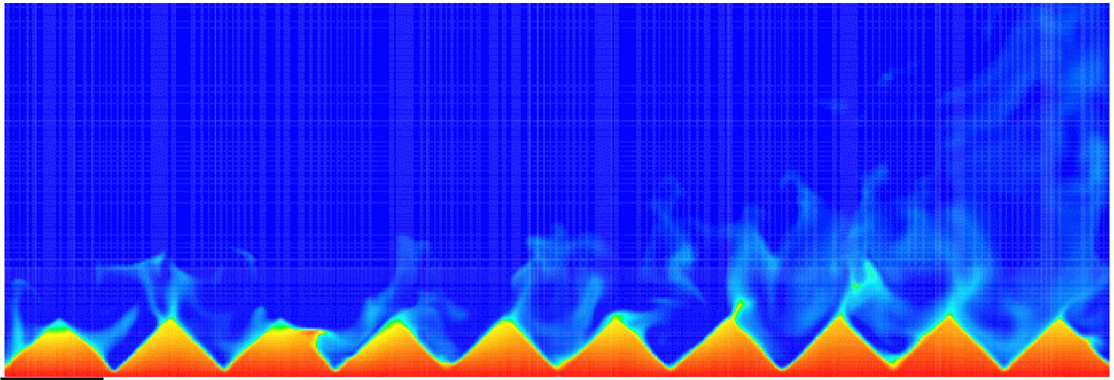
while it results $$\theta_p \approx \Delta Ra^{-1/8}$$ for constant temperature

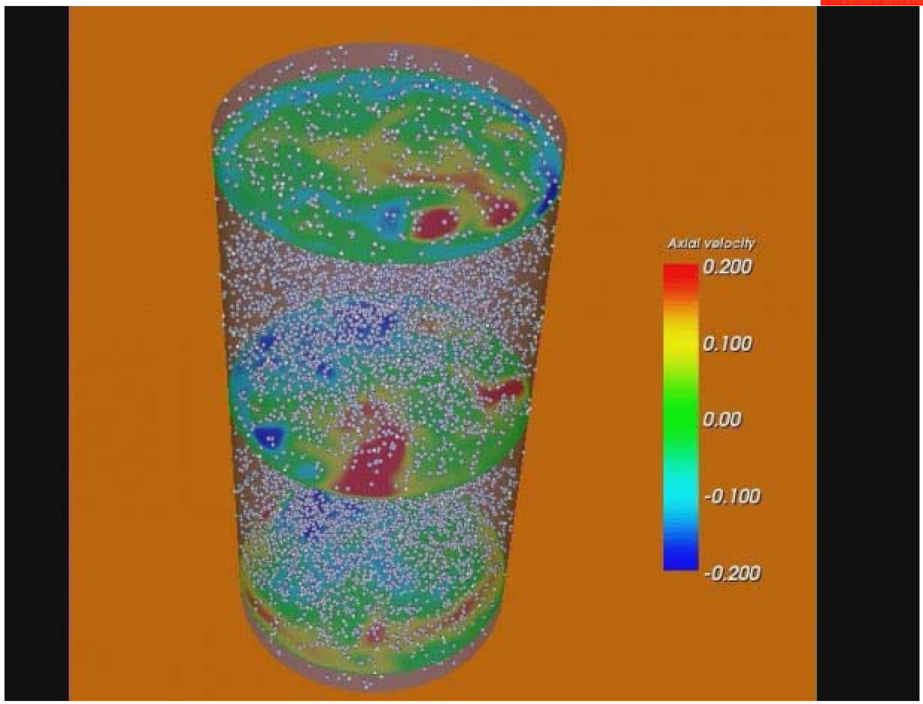
Beyond Rayleigh-Bénard convection

Fully non-Boussinesq turbulent thermal convection *(Sameen et al. 2008)* *(Ahlers et al. 2006, Sugiyama et al. 2007, 2008)*

Turbulent rotating thermal convection *(Oresta et al. 2007. Kunnen et al. 2008)*

Thermal convection with wall "roughness" *(Stringano et al. 2006)*





"Boiling"convection with gas bubbles *(Oresta et al. 2009)*

Closure

15 years ago DNS of turbulent Rayleigh-Bénard convection was out of reach of computers.

"most of experiments …. are well beyond the capabilities of current computers so serious compromises are required if simulations are to contribute at all to the discussion."

E.D. Siggia, *Annu. Rev. Fluid. Mech.* (1994)

Nowadays computers are powerful enough to make DNS a valid alternative and a good complement to many experiments

"Direct numerical simulations (DNS) of Rayleigh-Bénard flow have several advantages in comparison to experiments ….."

G. Ahlers, S. Grossmann & D. Lohse, *Annu. Rev. Fluid Mech. (2009), Rev. Modern Phys (2009)*