REVERSE MATHEMATICS

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Abstract. In this series of three talks we will describe the program of reverse mathematics, its origins, it goals, and some of the latest results.

The questions of which axioms are necessary to do mathematics is of great importance in Foundations of Mathematics and is the main question behind the program of Reverse Mathematics. Reverse Mathematics deals with the system of second-order-arithmetic which is rich enough to be able to express an important fragment of classical mathematics. This fragment includes number theory, calculus, countable algebra, real and complex analysis, differential equations, separable metric spaces and combinatorics among others. Almost all of mathematics that can be modeled with, or coded by, countable objects can be done in second-order arithmetic.

The idea of Reverse Mathematics goes as follows. We start by fixing a basic system of axioms. The most commonly used basic system is called RCA_0 that essentially says that computable sets exists. Now, given a theorem of "ordinary" mathematics, the question we ask is what axioms do we need to add to the basic system to prove this theorem. It is often the case in Reverse Mathematics that we can show that certain axioms are necessary to prove a theorem by showing that the axioms follow from the theorem using the basic system. Because of this idea, this program is called Reverse Mathematics. Many different systems of axioms have been defined and studied. But a very interesting fact is that most of the theorems, whose proof-theoretic strength has been analyzed, have been proved equivalent over RCA_0 to one of five systems, that we will call the *main five systems*. Understanding why only these main five systems are so frequently equivalent to theorems of ordinary mathematics is a very intriguing question.

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