The University of Manchester

Centre for Interdisciplinary Computational and Dynamical Analysis (CICADA)

Manchester Institute for Mathematical Sciences (MIMS)

The London Mathematical Society

CICADA / MIMS workshop on Numerics for Control and Simulation

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Introduction

Systems analysis and design procedures often require the efficient and reliable solution of linear or quadratic matrix equations. This can be seen in model reduction methods based on solutions of Lyapunov/Stein equations and algorithms in control theory (e.g., for linear-quadratic control or H_∞ control) based on the solution of Stein and/or Riccati equations. For large-scale problems computing a full solution is not feasible and only approximate solutions of low-rank can be computed. Remarkably, the structure of the overall problem often dictates that low-rank approximations to solutions of the matrix equations are sufficient. The goals of this workshop are to examine the feasibility of using low-rank solutions in systems and control applications, discuss recent developments in algorithms for computing them, and review applications in model reduction and simulation of large scale systems.

http://www.maths.manchester.ac.uk/~chahlaoui/NCS09/NCS09.htm

All talks will take place in room G205 (ground floor) in the Alan Turing Building at the University of Manchester.

Organizers

- Dr. Younes Chahlaoui, Centre for Interdisciplinary Computational and Dynamical Analysis (CICADA)
- Prof. Nick Higham, School of Mathematics.

Speakers

- Prof. Peter Benner (TU Chemnitz, DE).
- Dr. Daniel Kressner (ETH Zurich, CH).
- Dr. Karl Meerbergen (K.U.Leuven, BE)
- Dr. Valeria Simoncini (Universita' di Bologna, IT)
- Prof. Danny C. Sorensen (Rice University, Texas, USA).
- Dr. Tatjana Stykel (TU Berlin, DE).
- Prof. Paul Van Dooren (Catholic University of Louvain, BE).

Programme

09.15--09.45 Arrivals and Reception

09.45--10.00 Opening remarks

Nick Higham

10.00--10.50 *ADI-based methods for algebraic Lyapunov and Riccati equations.*
Peter Benner

10.50--11.20

11.20--12.10 *Discrete Empirical Interpolation for Nonlinear Model Reduction.*
Danny Sorensen

12.10--13.30 Lunch to be served on premises

13.30--14.20 *A multimesh approach for $\mathcal{H}_2$ model reduction.*
Paul Van Dooren

14.20--15.10 *Projected matrix equations and their application in model reduction of descriptor systems.*
Tatjana Stykel

15.10--15.30

15.30--16.20 *Matrix Equations and Bivariate Function Approximation.*
Daniel Kressner

16.20--16.45 *Pade via Lanczos with multiple right-hand sides.*
Karl Meerbergen

16.45--17.10 *Advances in projection-type methods for the numerical solution of the Lyapunov equation.*
Valeria Simoncini

17.10--18.00 *Informal discussion*

18.30 Conference dinner (at Tai Pan)

Abstracts

ADI-based methods for algebraic Lyapunov and Riccati equations

Peter Benner

The efficient numerical solution of large-scale Lyapunov equations and algebraic Riccati equations (AREs) is of fundamental importance for the efficient implementation of a variety of model reduction methods as well as for solving (optimal) control and stabilization problems for large-scale control problems. In recent years, significant progress has been made for solving large-scale Lyapunov equations and AREs for sparse or data-sparse coefficient matrices. We will survey these developments and highlight in particular approaches based on the ADI iteration for Lyapunov equations with low-rank right-hand side. If used as Lyapunov solver within the Newton-Kleinman framework for AREs, the Newton-ADI method results. We will address recent approaches to improve on the convergence of the ADI iteration for Lyapunov equations by employing a cyclic Galerkin projection. We will discuss the same idea for the quadratic ADI method for AREs. Moreover, we will address two issues that arise often in practical applications: high-rank constant terms or indefinite quadratic terms in the ARE prevent the application of the usual Newton-Kleinman iteration: in both situations, the Lyapunov equations to be solved in each iteration step have high-rank right-hand side. We will present recent ideas to overcome these difficulties.

Parts of this presentation are based on joint work with Ninoslav Truhar, Ren-Cang Li, Jens Saak, and Martin Köhler.

Discrete Empirical Interpolation for Nonlinear Model Reduction

Danny C. Sorensen

A dimension reduction method called Discrete Empirical Interpolation (DEIM) will be presented and shown to dramatically reduce the computational complexity of the popular Proper Orthogonal Decomposition (POD) method for constructing reduced-order models for unsteady and/or parametrized nonlinear partial differential equations (PDEs). In the presence of a general nonlinearity, the standard POD-Galerkin technique reduces dimension in the sense that far fewer variables are present, but the complexity of evaluating the nonlinear term remains that of the original problem.

I will describe DEIM as a modification of POD that reduces the complexity as well as the dimension of general nonlinear systems of ordinary differential equations (ODEs). It is, in particular, applicable to ODEs arising from finite difference discretization of unsteady time dependent PDE and/or parametrically dependent steady state problems. Our contribution is a greatly simplified description of Empirical Interpolation in a finite dimensional setting. The method possesses an error bound on the quality of approximation. An application of DEIM to a finite difference discretization of the 1-D FitzHugh-Nagumo equations is shown to reduce the dimension from 1024 to order 5 variables with negligible error over a long-time integration that fully captures non-linear limit cycle behavior. We also demonstrate applicability in higher spatial dimensions with similar state space dimension reduction and accuracy results.

A multimesh approach for $\mathcal{H}_2$ model reduction

Paul Van Dooren

We describe a multimesh approach for model reduction of large scale dynamical systems, modelled via state-space systems $\{A, B, C\}$ of the type

$$\begin{cases} \dot{x}(t) = A\,x(t)+B\,u(t) \\ y(t) = C\,x(t), \end{cases}$$

(with input $u(t) \in \mathbb{R}^m$, state $x(t) \in \mathbb{R}^N$ and output $y(t) \in \mathbb{R}^p$), obtained from a finite element discretization constructed on a fine mesh. The basic step of the algorithm is a fixed point iteration used to solve the $\mathcal{H}_2$ model reduction problem. This iteration requires the solution of a pair of Sylvester equations defined in terms of the state space equations and a current low order approximation $\{\widehat{A}, \widehat{B}, \widehat{C}\}$ of the dynamical system.

We show how this fixed point can be efficiently obtained using a multilevel approach where the fixed point iteration is applied only a few steps on each grid level. We illustrate these ideas on the construction of low order models for a particular example of convection diffusion equations.

Joint work with S. Melchior and V. Legat.

A Projected matrix equations and their application in model reduction of descriptor systems

Tatjana Stykel

We study projected Lyapunov, Sylvester, Lur'e and Riccati matrix equations. Such equations arise in many control problems for descriptor systems including stability analysis and balancing-related model reduction. We present the efficient algorithms based on the ADI iteration for the projected Lyapunov equations and Newton's method for the projected Riccati equations. Low-rank versions of these methods are also considered. The efficiency of our algorithms and their use in balanced truncation model reduction of large-scale descriptor systems are demonstrated on several practical problems.

Matrix Equations and Bivariate Function Approximation

Daniel Kressner

Balanced truncation model reduction for linear-time invariant control systems requires the solution of two Lyapunov equations

$$A Z_c + Z_c A^T = -BB^T, \quad A^T Z_o + Z_o A = -C^T C. \tag{0.1}$$

These matrix equations are intimately connected to the following approximation problem: Given the biivariate function $\frac{1}{(x+y)}$, find analytic functions $f_1, \ldots, f_k$ such that

$$\frac{1}{x+y} \approx \sum_{j=1}^{k} f_j(x) f_j(y).$$

For example, restricting f_j to be a polynomial yields an upper bound on the convergence of certain Krylov subspace methods for solving (1.1). Admitting f_j to be an exponential yields considerably sharp estimates on the singular value decay of the solutions.
The purpose of this talk is to present some known and some new results that arise from this connection, including their implications on numerical methods.

Padé via Lanczos with multiple right-hand sides

Karl Meerbergen

The solution of linear systems with a parameter is an important problem in engineering applications, especially in structural dynamics, acoustics, and electronic circuit simulations and related model reduction methods such as Padé via Lanczos.
In this talk, we present a method for solving symmetric parameterized linear systems with multiple right-hand sides, based on the Lanczos method. We show that for this class of applications, a simple deflation method can be used.

Advances in projection-type methods for the numerical solution of the Lyapunov equation

Valeria Simoncini

In this talk we report on recent results on the implementation and convergence analysis of Krylov subspace-type methods for the numerical solution of the Lyapunov equation, when the coefficient matrix has large dimension. The matrix is assumed to be dissipative and generally nonsymmetric.