

Mini-course in Financial Mathematics - Contagion Models in Credit Risk

Professor Mark H.A. Davis (Imperial College London)

10:30 - 12:30p.m. : G108, Alan Turing Building

2:00 - 5:00p.m. : G209, Alan Turing Building

These lectures aim to give an account of mathematical techniques for credit risk models where there is contagion between the obligors, i.e. default of one party either directly causes default of other parties or (more commonly) changes other parties' risk of default. While various approaches are possible, the treatment here concentrates on 'reduced-form' models based on Markov chains. We argue that such models provide a flexible and computationally efficient framework. Subsidiary but important themes are the role of information (i.e., whether various factors influencing default risk are observable or 'latent'), and changes of measure, either from 'physical' to 'risk neutral' and vice versa or, in the context of econometric studies, computation of likelihood functions for parameter estimation.

We start with a general discussion of joint distributions and copulas, mainly to point out that 'contagion' is in some sense already built into the copula concept. We then give a general formulation of the reduced-form model and a taxonomy of models distinguishing between factor, frailty and contagion models. The next section gives some background information about Markov processes, Markov chains and phasetype distributions as required for the subsequent sections. We then discuss four simple but effective Markov chain-based models with applications in counter party risk and credit risk for inhomogeneous and homogeneous portfolios.

We next move on to the 'subsidiary themes' mentioned above, which turn out to involve the relationship between Markov chain models and Piecewise-deterministic Markov Processes. Finally, we return to the Enhanced Risk homogeneous portfolio model in the light of these themes.