MATH20212

General Information

- Title: Algebraic Structures 2
- Unit code: MATH20212
- Credit rating: 10
- Level: 2
- Pre-requisite units: MATH21201
- Co-requisite units:
- School responsible: Mathematics
- Member of staff responsible: Mike Prest

Unit specification

Aims

The programme unit aims to introduce quotient structures and their connection with homomorphisms in the context of rings and then again in the context of groups; present further important examples of groups and rings and develop some of their properties with particular emphasis on polynomial rings, factorisation in rings and group actions.

Brief description

This course builds on Algebraic Structures 1, which is a prerequisite, and continues the strong emphasis on examples.
The construction of quotient objects and the relationship with homomorphisms is one of the main themes. This will be introduced in the context of rings, then used to construct roots of polynomials in extension fields. Factorisation in polynomial rings and rings of integers of number fields will also be studied in the first part.
The second part will begin by developing further properties of key examples, such as permutation groups, and will emphasise actions of groups. Then the construction of quotient objects and the connection with homomorphisms in the context of groups will be developed and illustrated by many examples.

Intended learning outcomes

On completion of this unit successful students will be able to:

- demonstrate their understanding of the quotient construction in theoretical terms and also in particular contexts;
- demonstrate their understanding of how to produce roots of polynomials in extension fields and be able to compute in such fields;
- solve a range of problems which require understanding of groups and rings;
- apply theoretical results to computations in particular examples of groups and rings.

Future topics requiring this course unit

Most, possibly all, algebra courses in years 3 and 4.

Syllabus

Part I: Rings
1. Definitions and examples (partly review): domains, fields and division rings; nilpotent and idempotent elements, products of rings; (many) examples; with students gaining familiarity with the ideas and examples through attempting exercises. [3]
2. Isomorphisms and homomorphisms (of rings): what is preserved and reflected; kernel of a homomorphism, ideals; principal ideals, operations on ideals. [3]
3. The quotient construction (for rings): the construction and connection with homomorphisms; maximal ideals; ideals of the quotient ring; examples. [2]
4. Polynomial rings and unique factorisation: unique factorisation domains, principal ideal domains and euclidean domains, with emphasis on rings of polynomials and rings of integers in number fields; construction of ring of fractions of a domain; tests for irreducibility, Gauss' Lemma. [3]
5. Constructing roots of polynomials: construction of extension fields; examples, including finite fields; algebraically closed fields. [3]

Part II: Groups
6. Definitions and examples (partly review): (sub)groups; G-sets, orbits and stabilisers; conjugacy classes, conjugacy of permutations; (many) examples; with students gaining familiarity with the ideas and examples through attempting exercises. [3
7. Isomorphisms and homomorphisms (of groups): what is preserved and reflected; kernel of a homomorphism, normal subgroups, products of groups; examples. [2]
8. The quotient construction (for groups): the construction and its connection with homomorphisms; (normal) subgroups of the quotient group; examples; groups of small order. [3]

Textbooks

J.B. Fraleigh, A First Course in Abstract Algebra , (any edition: the library has many copies) Addison-Wesley (recommended but not essential.

Also similar bookes like:

R.B.J.T. Allenby, Rings, Fields and Groups: an Introduction to Abstract Algebra , Addison-Wesley

Learning and teaching processes

Two lectures and one examples class each week

Assessment

Coursework Weighting within unit 20%

2 hours end of semester examination; Weighting within unit 80%

Arrangements