



MATH41431 - 2006/2007

General Information

- Title: Calculus on Manifolds
- Unit code: MATH41431
- Credits: 15
- Prerequisites: Vector calculus
- Co-requisite units: MATH30009 or MATH41009
- School responsible: Mathematics
- Member of staff responsible: Dr Ted Voronov (MSS P.5, Tel 63682)

Specification

Aims

Brief Description of the unit

Manifolds are one of the main notions of modern geometry and topology. They are used everywhere in mathematics and its applications. Roughly, manifolds are geometrical objects that can be endowed with coordinates, so that using these coordinates one can apply differential and integral calculus. Thus manifolds are arguably the most natural objects where calculus can be developed. On the other hand, the theory of manifolds provides for calculus and its applications a powerful 'invariant' geometrical language.

Learning Outcomes

Future topics requiring this course unit

Syllabus

1. Manifolds and smooth maps: examples and definition.
2. Tangent bundle, vectors and tensors. Differential forms as skew-symmetric tensors and their algebra. Manifolds as surfaces in Euclidean space (statement). Whitney Theorem: statement and basic idea of proof (modulo Saard's Theorem).
3. Commutators of vector fields. Lie derivative. Exterior differential of differential forms. Cartan formula and its generalization.
4. Topology induced by manifold structure. Compactness and connectedness: recollection and examples. Partition of unity: statement and main ideas of the proof. Proof that a compact manifold can be embedded into Euclidean space.
5. Orientation. Manifolds with boundary. Integration of differential forms. Stokes Theorem.
6. De Rham cohomology: definition. Examples of non-trivial cohomology classes. Homotopy invariance. Poincare Lemma. Examples of calculation of de Rham cohomology. De Rham Theorem (statement and idea of proof).

Textbooks

Teaching and learning methods

Two lectures per week with additional reading plus one weekly examples class.

Assessment

- Coursework; Weighting within unit 13%
- 3 hours end of semester examination; Weighting within unit 87%

Arrangements