MATH31412/MATH41412 - 2006/2007

General Information

- Title: Classical Integrable Systems
- Unit code: MATH31412/MATH41412
- Credits: 10 (MATH31412), 15 (MATH41412)
- Prerequisites: First year ordinary differential equations and matrix theory
- Co-requisite units:
- School responsible: Mathematics
- Member of staff responsible: Dr Alexander Odesskii

Specification

Aims

Brief Description of the unit

The notion of an 'integrable' differential equation can be thought of as a generalization of a differential equation which has an explicit solution. Such equations possess special properties such as an infinite number of conservation laws. Informally they are the opposite of equations which have a 'chaotic' behaviour. Integrable differential equations are important in both pure mathematics and in applications. They are a rich source of interesting algebraic and geometric structures. The course describes various ways of making the idea of 'integrability' precise. We introduce the main mathematical structures used in integrability theory and study various examples of integrable equations. This course builds on the study of differential equations and matrix theory in the first year.
The level 4 version of this course unit will be made up out of the level 3 version together with some additional reading material.

Learning Outcomes

Future topics requiring this course unit

Syllabus

1. Introduction. Idea of integrability on an informal level.
2. Hamiltonian mechanics. Commuting hamiltonians. Liouville integrability. The bi-hamiltonian approach to the construction of integrable systems.
3. Korteveg-de Vries equation. Lax pairs. Differential operators. Formal Pseudo-differential operators in one variable.
4. Other examples of equations with Lax pairs. Solitons. Backlund transformations.
5. Survey of definitions of integrability. Exact solvability. The Painleve test. Commuting flows and conservation laws. Hirota's bilinear equations.

Textbooks

What is integrability? Edited by V. E. Zakharov. Springer Series in Nonlinear Dynamics. Springer-Verlag, Berlin, 1991

Newell, Alan C. Solitons in mathematics and physics. CBMS-NSF Regional Conference Series in Applied Mathematics, 48. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1985.

Teaching and learning methods

Two lectures per week plus one weekly examples class.
Additional reading for the level 4 version.

Assessment
MT31412: 2 hour end of semester examination: weighting within unit 100%
MT41412: 3 hour end of semester examination: weighting within unit 100%

Arrangements