MANCHESTER 1824

MATH41422 - 2006/2007

General Information

- Title: Geometry of Fibre Bundles
- Unit code: MATH41422
- Credits: 15
- Prerequisites: MATH30009 or MATH41009, MATH31431 or MATH41431
- Co-requisite units:
- School responsible: Mathematics
- Member of staff responsible: Dr Ted Voronov (Mathematics and Social Sciences P.5, Tel: 63682)

Specification

Aims

Brief Description of the unit

This course unit covers the main notions of modern differential geometry, such as connection and curvature. It builds on the course unit MATH31431/Math41431 Calculus on Manifolds. Fibre bundles make a natural language for describing various 'fields' in geometry and its applications, such as vector fields or other fields appearing in physics. They are manifolds (or, more generally, topological spaces although this course will restrict attention to the differentiable case) with a special structure: they locally look like the product of a piece of one space called the base with another space called the fibre. A good example is the Möbius band which locally looks like a cylinder, the product of a circle with an interval, but as a whole space is twisted. A 'field' associates to each point in the base a point in the fibre. In order to differentiate it we need an extra structure known as a connection or covariant derivative. It often comes naturally in examples such as surfaces in Euclidean space. In this case a covariant derivative of tangent vectors can be defined can be defined as the usual derivative in the Euclidean space followed by orthogonal projection onto the tangent plane. The curvature of a connection in a fibre bundle is a new phenomenon which does not exist for the derivative of ordinary functions. It generalizes the 'internal' curvature of a surface discovered by Gauss which implies that it is impossible to map a region of a sphere onto a flat surface preserving distances. The course unit revises classical differential geometry of curves and surfaces, considers applications, and touches on the topology of fibre bundles. The level 4 version of this course unit will be made up out of the level 3 version together with some additional material provided by additional lectures and/or reading.

Learning Outcomes

Future topics requiring this course unit

Syllabus

1. Fibre bundles: definition and examples. Particular case: vector bundles. Transition functions. Cocycle property. Pull-back of fibre bundles.
2. Operations with fibre bundles. Metric on a vector bundle.
3. Covariant derivative. Examples. Parallel transport.
4. Covariant derivative for the tangent bundle. Levi-Civita Theorem. Goedesics. Relation with the least action principle.
5. Curvature. Examples. Bianchi identity. Properties of the Riemann tensor.
6. Curves and surfaces in R^3. Derivation formulae. Theorema Egregium.
7. Homotopy propery of vector bundles. Embedding into a trivial vector bundle (statement and proof). Classifying spaces for vector bundles.
8. Differential-geometric characteristic classes: construction. Examples. Classification theorem (statement and proof). Relation with classifying spaces. Gauss-Bonnet Theorem for surfaces in R^3 and Gauss-Bonnet-Chern Theorem for Riemannian manifolds (statement).

Textbooks

Teaching and learning methods

Two lectures per week with additional reading plus one weekly examples class.

Assessment
: End of semester examination (3 hours) 100%.

Arrangements