MATH20512

General Information

- Title: Mechanics and Waves
- Unit code: MATH20512
- Credit rating: 10
- Level: 2
- Pre-requisite units: MATH20401 or MATH20411
- Co-requisite units:
- School responsible: Mathematics
- Member of staff responsible: Dr James Montaldi

Unit specification

Aims

The programme unit aims to

Brief description

The course aims to develop an understanding of how Newton's laws of motion can be used to describe the motion of particles and to introduce the rudiments of wave mechanics.

The first part of the unit concerns the general description and analysis of the motion of particles acted on by forces. Assuming a basic familiarity with Newton's laws of motion and their application in simple situations, we shall develop the advanced techniques necessary for the study of more complicated systems. We shall also consider the beautiful generalizations of Newton's equations due to Lagrange and Hamilton, which allow for simplified treatments of many interesting problems and which provide the foundation for the modern understanding of dynamics. The second theme within the course aims to characterise and elucidate some of the physical properties of important types of wave motions and their mathematical descriptions . Waves occur everywhere in the physical world: from the small scale (e.g. ultrasonic waves in biomedical imaging) to the very large (such as ocean waves). This introductory course discusses a number of basic ideas fundamental to wave dynamics and offers a range of simple applications. The course is a useful primer to third and fourth level course units in physical applied mathematics.

Intended learning outcomes

On completion of this unit successful students will be able to:

- model simple mechanical systems, both in inertial and rotating frames, using Lagrange's equations;
- analyze the dynamics of systems near equilibrium; find the normal modes of oscillation;
- apply the principle of least action to relate Hamiltonian and Lagrangian approaches
- understand the basics of waves in infinite 1-dimensional domains; determine the group and phase velocities of such waves

Future topics requiring this course unit
Third and fourth level course units in physical applied mathematics.

Syllabus

1. Newtonian Mechanics of Systems of Particles. Review of Newton's laws; centre of mass; basic kinematic quantities: momentum, angular momentum and kinetic energy; circular motion; 2-body problem; conservation laws; reduction to centre of mass frame. [5]
2. Small Oscillations. Particle in a potential well; coupled harmonic oscillators; normal modes; wave equation on finite string and Fourier modes. [3]
3. Rotating Frames and the Rigid Body. Centrifugal and Coriolis forces; moments of inertia, parallel axes theorem; free rigid body rotation, Euler's equations. [4]
4. Lagrangian formulation of mechanics. Generalized coordinates; constraints; cyclic variables; examples. [4]
5. Hamiltonian formulation. Principle of least action; equivalence with Lagrangian formulation; equilibria; stability; conserved quantities. [3]
6. Waves revisited. D'Alembert's solution for the infinite string, group velocity, phase velocity, energy and dispersion. [3]

Textbooks

Learning and teaching processes
Two lectures and one examples class each week

Assessment
Coursework Weighting within unit 20%
2 hours end of semester examination; Weighting within unit 80%

Arrangements