MATH20401

General Information

- Title: Partial Differential Equations and Vector Calculus
- Unit code: MATH20401
- Credit rating: 20
- Level: 2
- Pre-requisite units: MT1121 or MT1131, MT1222 or MT1232
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Dr David Harris, Prof David Silvester, Dr Mike Simon

Unit specification

Aims

This course introduces students to (i) analytical and numerical methods for solving partial differential equations (PDEs), (ii) concepts and methods of Vector calculus. It builds on the first year core applied mathematics courses to develop more advanced ideas in differential and integral calculus.

Brief description

The first half of the course consists of an introduction to the important topics of Fourier series, partial differential equations and analytical methods for solving first and second order PDEs (including Laplace's, heat and wave equations). The second half of the course begins with an introduction to the specialist topic of numerical analysis by considering finite difference methods for solving differential equations. It continues with several topics in vector calculus, including line, surface and volume integrals and concludes with the divergence, Green's and Stokes' theorems and an introduction to tensors. The methods employed in the course will prove essential for all of the applied mathematics and numerical analysis options in the remaining semesters of the BSc and MMath degree programmes.

Intended learning outcomes

On completion of this unit successful students will be able to:

- Convert Cartesian coordinates into cylindrical and spherical polar coordinates and solve problems expressed in these coordinate systems
- Understand functions of several variables, their partial differentiation, integration, and their geometrical interpretation
- Understand the basic concept of orthogonal functions
- Compute Fourier series, Fourier sine series and Fourier cosine series of piecewise continuous functions
- Solve first order parial differential equations via the method of characteristics
- Recognise classical PDEs describing physical processes such as diffusion, wave propagation and electrostatics
- Choose appropriate boundary and initial conditions for PDEs
- Classify second-order PDEs as elliptic, hyperbolic or parabolic

- Solve analytically, using the method of separation of variables, the heat and wave equations (in one space variable) and Laplace's equation (in two space variables) on rectangular and circular domains
- Solve numerically, via finite difference schemes, the heat equation in one space variable
- Solve numerically, convection-diffusion equations using upwind finite differencing
- Compute elements of surface and volume in different coordinate systems
- Evaluate line, surface and volume integrals over domains using various coordinate systems
- Use grad, div and curl operator notation and relate key identities to properties of vector fields
- Understand, and use the Divergence, Green's and Stokes' theorems
- Understand the concept of a tensor and be able to use indicial notation and the summation convention

Future topics requiring this course unit
The material in this course unit is essential for all applied mathematics options in subsequent semesters.

Syllabus

1. Introductory material [3 lectures] Cartesian, curvilinear coordinates (including cylindrical, spherical polars). functions of several variables and partial derivatives. Surfaces, unit vectors, elements of surface/volume. Integrals of functions of several variables.
2. Fourier Series [4 lectures] Periodic functions. Orthogonality. Fourier coefficients and Fourier series: Fourier's theorem. Parseval's theorem.
3. Partial Differential Equations [13 lectures] Linearity, homogeneity, order. Classification: elliptic, parabolic, hyperbolic equations. Method of characteristics for first order PDEs. Quasi-linear equations. PDE's: problems reducible to ODE's. Second order equations, Laplace's, diffusion and wave equations. Separation of variables. Solution of second order ordinary differential equations (e.g. Bessel's equation) by series expansion. Initial value problems for heat and wave equations. Eigenfunction series and normal modes. Laplace's equation separated in Cartesian and plane polar coordinates; applications to heat conduction and electrostatics.
4. Numerical Aanalysis of pde's [13 lectures] Finite diference methods. Heat equation in one space variable. Explicit and implicit schemes, truncation error, stability and convergence. Crank-Nicholson scheme. Convection-diffusion problems, upwind differencing. The Poisson equation in two dimensions. Maximum principle and sketch of convergence analysis.
5. Vector Calculus [11 lectures] Line, surface and volume integrals. Scalar and vector fields: differential and integral calculus. Field lines and field surfaces, the general field surface of a vector field. Grad, div and curl in Cartesian and curvilinear coordinates. Identities. Divergence, Green's and Stokes' theorems. Indicial notation, summation convention. Zero, first, second and higher order tensors.

Textbooks
To be notified later

Learning and teaching processes
Four lectures and one examples class each week

Assessment
Coursework Weighting within unit 20%
3 hours end of semester examination; Weighting within unit 80%

Arrangements