MATH46121/MATH66121 - 2007/2008

General Information

- Title: Applied Dynamical Systems
- Unit code: MATH46121/66121
- Credits: 15
- Prerequisites: Familiarity with MATLAB.
- Co-requisite units: MATH46101 *Numerical Linear Algebra* if not familiar with MATLAB.
- School responsible: Mathematics
- Members of staff responsible: Dr. Jeremy Huke

Specification

Aims

To develop a basic understanding dynamical systems theory, particularly those aspects important in applications. To describe and illustrate how the basic behaviours found in dynamical systems may be recognized and analyzed.

Brief Description of the unit

Dynamical systems theory is the mathematical theory of time-varying systems; it is used in the modelling of a wide range of physical, biological, engineering, economic and other phenomena. This module presents a broad introduction to the area, with emphasis on those aspects important in the modelling and simulation of systems. General dynamical systems are described, along with the most basic sorts of behaviour that they can show. The dynamical systems most commonly encountered in applications are formed from sets of differential equations, and these are described, including some practical aspects of their simulation. The most regular kinds of behaviour---equilibrium and periodic---are the most easy to analyze theoretically; linearization about such trajectories are discussed (for periodic behaviour this is done using the *Poincaré map*.)

Much more complex behaviours, including chaos, may be found; these are described by means of their attractors. The linearization approach can be extended to these, and leads to the concept of Lyapunov exponents.

In applications it is often important to know how the observed behaviour changes with changes in the system parameters; such changes can often be sudden, but frequently conform to one of a relatively small number of scenarios: the study of these forms the subject of bifurcation theory. The simplest bifurcations are discussed.

Learning Outcomes

On successful completion of this course unit students will

- understand the general concept of a dynamical system, and the significance of dynamical systems for modelling real world phenomena;
- be able to analyze simple dynamical systems to find and classify regular behaviour;
- appreciate some of the more complex behaviours (including chaotic), and understand some of the features of the attractors characterizing such behaviour;
- be familiar with some of the simpler bifurcation scenarios, and how they can be analyzed.

Future topics requiring this course unit

None.

Syllabus

1. **Basics.** Basic concepts of dynamical systems: states, state spaces, dynamics. Discrete and continuous time systems. [1 lecture]
   Some motivating examples: (discrete): simple population models, numerical algorithms; (continuous): chemical and population kinetics, mechanical systems, electronic and biological oscillators. [1]

2. **Basic features of dynamical systems.** Trajectories, fixed points, periodic orbits, attractors and basins. Autonomous and non-autonomous systems. Phase portraits in the plane and higher dimensions; examples of phase portraits of 2-d and 3-d systems. [2]
3. **Ordinary differential equations.** Systems of first order ordinary differential equations; initial value problems, existence and uniqueness of solutions. Flows. [2]
4. **Numerical solution of ODEs.** Single and multistep methods; explicit and implicit schemes; stiff systems. [3]
5. **Equilibria and linearization.** Fixed and equilibrium points and their linearization; classification and the Hartman-Grobman theorem; examples in 2-d and 3-d. Computing equilibrium points. [3]
6. **Periodic orbits and linearization.** Poincarésections and the Poincaré map. Linearization and characteristic multipliers of periodic orbits, and stability; examples. Computing periodic orbits. [3]
7. **Invariant manifolds.** Invariant manifolds for equilibrium points and periodic orbits. Computing one-dimensional invariant manifolds. [3]
8. **Attractors and long-term behaviour.** ω-limit sets and long term behaviour. Chaotic attractors; illustrative examples. Lyapunov exponents and their computation. [4]
9. **Bifurcations in 1-d maps.** Fold, transcritical, period-doubling and pitchfork bifurcations; bifurcation diagrams, examples. Structural stability. Bifurcation theorems and imperfect bifurcations. [3]
10. **Bifurcations of flows.** Hopf bifurcations, global bifurcations; examples. Computing bifurcation diagrams by continuation. Bifurcation routes to chaos, crises, chaotic transients. [5]

Textbooks

- Stephen~H. Strogatz, *Nonlinear Dynamics and Chaos*, Perseus Books, Cambridge, MA, USA, 1994.
- Kathleen T. Alligood, Tim D. Sauer and James A. Yorke, *Chaos: An Introduction to Dynamical Systems*, Springer-Verlag, New York, NY, USA, 1996.
- Thomas S. Parker and Leon O. Chua, *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, New York, NY, USA, 1989.
- Frank C. Hoppensteadt, *Analysis and Simulation of Chaotic Systems*, Springer-Verlag, New York, NY, USA, second edition, 2000.
- Stephen Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer-Verlag, New York, NY, USA, second edition, 2003.

Teaching and learning methods

30 lectures (two or three lectures per week) with a weekly examples class. In addition should expect to spend at least seven hours each week on private study..

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements