MATH47112 - 2007/2008

General Information

- Title: Brownian Motion
- Unit code: MATH47112
- Credits: 15
- Prerequisites: MATH20722 *Classical Probability* or equivalent.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. Goran Peskir

Specification

Aims

The unit aims to provide the basic knowledge necessary to pursue further studies/applications where Brownian motion plays a fundamental role (e.g. Financial Mathematics).

Brief description of the course unit

Brownian motion is the most important stochastic process. It was observed by Brown in 1828 and explained by Einstein in 1905. A more accurate model based on work of Langevin from 1908 was introduced by Ornstein and Uhlenbeck in 1930. The assumption of stationary independent increments made by Einstein in 1905 has had a profound influence on the development of probability theory in the 20th century. The course unit presents basic facts and ideas of Brownian motion paying particular attention to the issues of dynamics.

Learning outcomes

On successful completion of this course unit students will

- understand the concept of Brownian motion and diffusion processes;
- understand the exact relation of these processes to PDEs with boundary conditions;
- be able to apply these relations to exploit the interplay between probability and analysis.

Future topics requiring this course unit

None.

Syllabus

1. The heat equation (Fourier's law). [1 lecture]
2. The diffusion equation (Fick's law). [1]
3. Einstein's derivation of the diffusion equation (stationary independent increments). [2]
4. The Wiener process (position of a Brownian particle). [6]
5. The Ornstein-Uhlenbeck process (velocity of a Brownian particle). [2]
6. Strong Markov property (starting afresh at stopping times). [2]
7. Diffusion processes (scale function, speed measure, infinitesimal operator). [8]
8. Boundary classification (regular, exit, entrance, natural). [2]
9. The Kolmogorov forward and backward equations. [2]
10. Probabilistic solutions of PDEs (elliptic and parabolic). [6]
11. Optimal stopping, free boundary problems, the American option problem. [2]

12. Optimal stochastic control, the Hamilton-Jacobi-Bellman equation, the optimal consumption-investment problem. [2]

Textbooks

- Rogers, L. C. G. and Williams, D., *Diffusions, Markov Processes and Martingales*, Vol. 1 & 2, Cambridge University Press 2000.
- Revuz, D. and Yor, M., *Continuous Martingales and Brownian Motion*, Springer 1999.
- Karatzas, I. and Shreve, S. E., *Brownian Motion and Stochastic Calculus*, Springer 1991.
- Karlin, S. and Taylor, H. M., *A Second Course in Stochastic Processes*, Academic Press 1981.
- Nelson, E., *Dynamical Theories of Brownian Motion*, Princeton University Press 1967.

Teaching and learning methods

Three lectures and one examples class each week. In additiona students should expect to spend at least six hours each week on private study for this course unit.

Assessment

End of semester examination: three hours weighting 100%.

Arrangements