MATH32012 - 2007/2008

General Information

- Title: Commutative Algebra
- Unit code: MATH32012
- Credits: 10
- Prerequisites: MATH20201*Algebraic Structures 1*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Gennady Puninskiy

Specification

Aims

The course unit will deepen and extend students' knowledge and understanding of algebra. By the end of the course unit the student will have learned more about familiar mathematical objects, will have acquired various computational and algebraic skills, will have seen how the introduction of structural ideas leads to the solution of mathematical problems and will have built a solid foundation for any further study of algebra and algebraic structures.

Brief Description of the unit

Polynomials are familiar mathematical objects which play a part in virtually every branch of the subject, e.g. solution of quadratic equations, the use of quadratic forms to study maxima and minima of functions, approximation of functions by Taylor or Chebyshev polynomials, the characteristic polynomial of a matrix, etc. In algebra you have probably met problems involving factoring polynomials and finding the gcd of two polynomials (in one variable) using the Euclidean algorithm.

Historically the study of solutions of polynomial equations (algebraic geometry) and the study of symmetries of polynomials (invariant theory) were a major source of inspiration for the vast expansion of algebra in the 19th and 20th centuries.

In this course the algebra of polynomials in *n* variables over a field of coefficients is the basic object of study. The course covers recent advances in the subject with important applications to computer algebra, together with a selection of more classical material.

Learning Outcomes

On successful completion of this course unit students will be able to demonstrate

- facility in dealing with polynomials (in one and more variables);
- understanding of some basic ideal structure of polynomial rings;
- ability to compute generating sets and Gröbner bases for such ideals;
- ability to relate work with polynomials to the context of rings, ideals, and other algebraic structures;
- in particular, ability to solve problems relating to the factorisation of polynomials, irreducible polynomials and extension fields;
- facility in dealing with symmetric and alternating polynomials;
- ability to relate work with symmetric functions to the context of invariants of finite groups.

Future topics requiring this course unit

None.

Syllabus

1. **Computing with Polynomials:** Polynomials in two or more variables, ideals in polynomial rings, monomial ideals, orderings on monomials, reduction and remainders, Gröbner bases, Hilbert's basis theorem, S-polynomials and Buchberger's algorithm.

2. **The Algebra of Polynomials:** Unique factorisation, irreducible polynomials, quotient rings, prime and maximal ideals, field extensions.
3. **Symmetries of Polynomials:** Symmetric polynomials, elementary symmetric functions, relation between roots and coefficients of one-variable polynomial, the fundamental theorem on symmetric functions, alternating polynomials and determinants, polynomial invariants of finite groups.

Textbooks

You are recommended to own at least one good general algebra textbook such as the first two below. These do not cover Gröbner bases (a relatively new topic).

The book by Cox, Little and O'Shea covers all the course material and much more. The last book below is a new textbook which combines Gröbner bases with more traditional material.

- R.B.J.T. Allenby, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, Edward Arnold, 0-3405-4440-6.
- J.B. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley,1994, 0-201-59291-6.
- D. Cox, J. Little and D. O'Shea, *Ideals, Varieties and Algorithms*, Springer 1992 (2nd edition) 1997, 0-387-94680-2.
- Niels Lauritzen, *Concrete Abstract Algebra*, Cambridge University Press 2004, 0-5215-3410-0.

Teaching and learning methods

Two lectures and one examples class each weekts. In addition students should expect to spend at least four hours each week on private study for this course unit.

Course notes will be provided, as well as examples sheets and solutions. The notes will be concise and will need to be supplemented by your own notes taken in lectures, particularly of worked examples.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements