MATH46142 - 2007/2008

General Information

- Title: Finite Element Analysis
- Unit code: MATH46142
- Credits: 15
- Prerequisites:
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Prof. David Silvester

Specification

Aims

To give an understanding of the theoretical basis underlying finite element approximation methods. To provide students with the technical tools enabling them to solve practical PDE problems, especially those that arising in modelling steady incompressible flow.

Brief Description of the unit

This course unit covers the numerical solution of linear elliptic partial differential equations (PDEs) using finite element approximation methods. Such methods are universally used to solve practical problems associated with physical phenomena in complex geometries. The emphasis is on assessing the accuracy of the approximation using a priori and a posteriori error estimation techniques. Practical issues will be illustrated with MATLAB using the IFISS toolbox and the P-IFISS toolbox.

Learning Outcomes

On successful completion of this course unit students will

- understand the concepts of weak and classical solutions of elliptic boundary value problems;
- understand the concept of piecewise polynomial approximation in two dimensions, and have an appreciation for the underlying error analysis;
- have an appreciation of the computational issues that arise when solving convection-diffusion problems;
- be familiar with the stability issues that arise when using so-called mixed approximation methods.

Future topics requiring this course unit

None.

Syllabus

1. **Basics.** Review of basic functional analysis concepts: norms, inner-products. Sobolev spaces. Weak derivatives. Lax-Milgram lemma. [2]
2. **Finite element methods for the Poisson equation.** Affine mappings. Linear, bilinear, quadratic and biquadratic approximation. Finite element assembly process. Properties of the discrete equation system. A priori error bounds: best approximation in energy, H^1 error bounds. H^2 regularity and singular problems. A posteriori error bounds. Local error estimators. Self adaptive refinement strategies. [12]
3. **Finite element methods for the convection-diffusion equation.** Well-posedness. Weak formulation. Galerkin approximation. The streamline-diffusion method. A priori and a posteriori error bounds. Self-adaptive refinement strategies for resolving layers. [6]

4. **Mixed finite element methods.** Saddle-point problems. Well-posedness. The Stokes equations. Potential flow equations for flow in heterogeneous media. Weak-formulation. Inf-sup stability. Stable and stabilized mixed approximation. A priori error bounds: optimal approximation in energy. A posteriori error bounds. [10]

Textbooks

- Howard Elman, David Silvester and Andy Wathen, *Finite Elements and Fast Iterative Solvers*, Oxford University Press, Oxford 2005, ISBN 0-19-852868-X (pbk).
- Dietrich Braess, *Finite Elements: Theory, Fast Solvers and Applications in Solid Mechanics*, Cambridge University Press, second edition, 2001, ISBN 0-521-01195-7 (pbk).
- Susanne C. Brenner and L. Ridgeway Scott, *The Mathematical Theory of Finite Element Methods*, Springer-Verlag, New York 1994.

Teaching and learning methods

30 lectures (two or three lectures per week) with a weekly examples class. In addition should expect to spend at least seven hours each week on private study..

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements