MATH41101 - 2007/2008

General Information

- Title: Gometric Cobordism Theory
- Unit code: MATH41101
- Credits: 15
- Prerequisites: MATH31051 *Introduction to Topology*
- Co-requisite units:
- School responsible: Mathematics
- Member of staff responsible: Prof. Victor Buchstaber

Specification

Aims

Cobordism theory gives an extremely powerful tool for the solution of geometrical problems by the methods of algebraic topology. The main goal of this course is to base the construction of cobordism theory on the differential geometry of smooth manifolds and to show some important applications of the theory.

Brief Description of the unit

The introductory parts of the course will parallel those of Milnor's *Topology from the Differential Viewpoint* (see below), and the subsequent lectures will work through as many of the following topics as time permits.

This course unit will complement the 2006/2007 course unit MATH40121 *Adavanced Algebraic Topology*. It should be of value to students who took that course unit but should also be accessible to students who did not. The notes for the earlier course unit are available.

Learning Outcomes

Future topics requiring this course unit

None.

Syllabus

1. **Smooth manifolds:**
   1.1 Basic definitions.
   1.2 Constructions on manifolds.
   1.3 Important examples.
2. **Bordism theory:**
   2.1 Bordism of manifolds.
   2.2 Bordism groups of a space.
   2.3 Axioms of a homology theory for bordism groups.
3. **Vector bundles:**
   3.1 Basic definitions.
   3.2 Constructions on vector bundles.
   3.3 Vector bundles with additional structure.
4. **Cobordism theory:**
   4.1 Orientation of maps.
   4.2 Cobordism groups of a manifold.
   4.3 Axioms of a cohomology theory for cobordism groups.
   4.4 Poincare duality.
5. **Thom spaces:**
   5.1 Basic definitions.
   5.2 The Pontrjagin-Thom construction.

5.3 Cobordism groups from a homotopical point of view.
5.4 Atiyah duality.
6. **Characteristic classes in cobordism theory:**
   6.1 Euler classes.
   6.2 Transfer maps.
   6.3 Stiefel-Whitney, Chern and Pontrjagin classes.
7. **Cohomological operations in cobordism theory:**
   7.1 The Landweber-Novikov algebra.
   7.2 tom Dieck-Steenrod operations.
   7.3 Important examples.
8. **Applications:**
   As time permits.

Textbooks

- John W. Milnor, *Topology from the Differentiable Viewpoint*, The University Press of Virginia, Charlottesville, 1978.
- Allen Hatcher, *Vector Bundles and K-theory*, version 2.0, January 2003, see home page of A. Hatcher.
- P.E.Conner, E.E.Floyd, *Differentiable Periodic Maps*, Springer-Verlag, 1964.
- Daniel Quillen, *Elementary proofs of some results of cobordism theory using Steenrod operations*, Advances in Mathematics 7, 29-56,1971.

Teaching and learning methods

Two lectures and an examples classs each week. In addition students should expect to spend at least seven hours on private study on this course unit.

Assessment

End of semester examination: three hours weighting 100%

Arrangements