MATH20101 - 2007/2008

General Information

- Title: Real and Complex Anaylsis
- Unit code: MATH20101
- Credit rating: 20
- Level: 2
- Pre-requisite units: MATH10101 or MATH10111, MATH10121 or MATH10131, MATH10242
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Mark Coleman and Prof. Richard Sharp

Unit specification

Aims

The programme unit aims to introduce the basic ideas of real analysis (continuity, differentiability and Riemann integration) and their rigorous treatment, and then to introduce the basic elements of complex analysis, with particular emphasis on Cauchy's Theorem and the calculus of residues.

Brief description

The first half of the course describes how the basic ideas of the calculus of real functions of a real variable (continuity, differentiation and integration) can be made precise and how the basic properties can be developed from the definitions. It builds on the treatment of sequences and series in MT1242. Important results are the Mean Value Theorem, leading to the representation of some functions as power series (the Taylor series), and the Fundamental Theorem of Calculus which establishes the relationship between differentiation and integration.

The second half of the course extends these ideas to complex functions of a complex variable. It turns out that complex differentiability is a very strong condition and differentiable functions behave very well. Integration is along paths in the complex plane. The central result of this spectacularly beautiful part of mathematics is Cauchy's Theorem guaranteeing that certain integrals along closed paths are zero. This striking result leads to useful techniques for evaluating real integrals based on the `calculus of residues'.

Intended learning outcomes

On completion of this unit successful students will be able to:

- understand the concept of limit for real functions and be able to calculate limits of standard functions and construct simple proofs involving this concept;
- understand the concept of continuity and be familiar with the statements and proofs of the standard results about continuous real functions;

- understand the concept of the differentiability of a real valued function and be familiar with the statements and proofs of the standard results about differentiable real functions;
- appreciate the definition of the Riemann integral, and be familiar with the statements and proofs of the standard results about the Riemann integral including the Fundamental Theorem of Calculus;
- understand the significance of differentiability for complex functions and be familiar with the Cauchy-Riemann equations;
- evaluate integrals along a path in the complex plane and understand the statement of Cauchy's Theorem and have seen an outline of the proof;
- compute the Taylor and Laurent expansions of simple functions, determining the nature of the singularities and calculating residues;
- use the Cauchy Residue Theorem to evaluate integrals and sum series.

Future topics requiring this course unit

Real analysis is needed in more advanced courses in analysis, functional analysis and topology and some courses in numerical analysis.

Complex analysis is needed for advanced analysis, geometry and topology, but also has applications in differential equations, potential theory, fluid mechanics, asymptotics and wave analysis.

Syllabus

Real analysis

1. **Limits.** Limits of real-valued functions, sums, products and quotients of limits. [5 lectures]
2. **Continuity.** Continuity of real-valued functions, sums, products and quotients of continuous functions, the composition of continuous functions. Boundedness of continuous functions on a closed interval. The Intermediate Value Theorem. The Inverse Function Theorem. [5]
3. **Differentiability.** Differentiability of real-valued functions, sums, products and quotients of continuous functions, Rolle's Theorem, the Mean Value Theorem, Taylor's Theorem. [5]
4. **Integration.** Definition of the Riemann integral, integrability of monotonic and continuous function, the Fundamental Theorem of Calculus, Improper integrals. [5]

Complex analysis

5. **The complex plane.** The topology of the complex plane, open sets, complex sequences and series, power series, and continuous functions. [4]
6. **Differentiation.** Differentiable complex functions and the Cauchy-Riemann equations. [2]
7. **Integration.** Integration along paths, the Fundamental Theorem of Calculus, the Estimation Lemma, Cauchy's Theorem. [4]
8. **Argument and Logarithm.** [2]
9. **Taylor and Laurent Series.** Cauchy's Integral Formula and Taylor series, Louiville's Theorem and the Fundamental Theorem of Algebra, zeros and poles, Laurent series. [5]
10. **Residues.** Cauchy's Residue Theorem, the evaluation of definite integrals and summation of series. [5]

Textbooks

Rod Haggerty, *Fundamentals of Mathematical Analysis*, Addison-Wesley, second edition 1993.
Ian Stewart and David Tall., *Complex Analysis*, Cambridge University Press, 1983.,

Learning and teaching processes

Four lectures and two examples classes each week.

Assessment

Coursework; Weighting within unit 20%

3 hours end of semester examination; Weighting within unit 80%

Arrangements

Notes

For the **Complex Analysis** part students will be expected to have access to a copy of the Complex Analysis book listed above.