



MATH32022 - 2007/2008

General Information

- Title: Reflection Groups
- Unit code: MATH32022
- Credits: 10
- Prerequisites: MATH10202 or MATH10212 *Linear Algebra*, MATH20201 *Algebraic Structures 1* and MATH20212 *Algebraic Structures 2*. MATH32001 *Group Theory* is highly desirable, but formally not essential. Actual prerequisites for taking the course are:
 1. a good knowledge of Linear Algebra, especially the theory of orthogonal transformations in real Euclidean spaces;
 2. the following basic notions of Group Theory: groups, the order of a finite group, subgroups, normal subgroups and factor (or quotient) groups, homomorphisms and isomorphisms, permutations (standard notation for them and rules for their multiplication), cyclic groups, action of a group on a set.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. [Alexandre Borovik](#)

Specification

Aims

This lecture course aims to introduce students to the classification and construction of finite reflection groups.

Brief Description of the unit

Finite reflection groups constitute one of the most important and frequently used classes of groups. The development of the theory in the course will be purely geometrical; the necessary facts from geometry (hyperplane arrangements, convex cones and polytopes) will be introduced as the need arises.

Learning Outcomes

On successful completion of this course unit students will

- know the classification of regular polytopes in three dimensions and their symmetry groups;
- know the basic examples of finite reflection groups in higher dimensions;
- understand the concept of a root system associated with a reflection group;
- understand the concept of a Coxeter diagram of a reflection group;
- understand the relations between root systems, finite reflection groups and Coxeter diagrams.

Future topics requiring this course unit

None.

Syllabus

1. Geometric background: affine Euclidean space, isometries, hyperplane arrangements, polyhedral cones. [4 lectures]
2. Reflections and their mirrors. [1]
3. Systems of mirrors and finite reflection groups. [1]
4. Dihedral groups. [1]
5. Root systems. [1]
6. Root systems of series A , B , C , D and their reflection groups. [2]
7. Chambers and chamber systems. [1]
8. Generation. [1]
9. The Coxeter complex. [1]
10. Residues. [1]
11. Generators and relations. [1]

12. The classification of finite reflection groups. [2]
13. Construction of root systems, the crystallographic condition. [1]
14. Orders of reflection groups. [1]
15. Reflection Groups in Three Dimensions [1]
16. The icosahedron. [1]

Textbooks

- M. A. Armstrong, *Groups and Symmetry*, Springer-Verlag 1988 and later editions.
- L. C. Grove and C. T. Benson, *Finite Reflection Groups*, Springer-Verlag 1984 and later editions.

Teaching and learning methods

Three classes each week which will include opportunities to discuss problems from the Problems Sheets.

Assessment

Mid-semester coursework: weighting 15%
Two hours end of semester examination: weighting 85%

Arrangements

Detailed lecture notes will be provided for this unit.