MATH32012 - 2008/2009

General Information

- Title: Commutative Algebra
- Unit code: MATH32012
- Credits: 10
- Prerequisites: MATH20201 *Algebraic Structures 1,* MATH20212 *Algebraic Structures 2.*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. M. Prest.

Specification

Aims

The course unit will deepen and extend students' knowledge and understanding of commutative algebra. By the end of the course unit the student will have learned more about familiar mathematical objects such as polynomials and algebraic numbers, will have acquired various computational and algebraic skills and will have seen how the introduction of structural ideas leads to the solution of mathematical problems.

Brief Description of the unit

The central theme of this course is factorisation (theory and practice) in commutative rings; rings of polynomials are our main examples but there are others, such as rings of algebraic integers.

Polynomials are familiar objects which play a part in virtually every branch of mathematics. Historically, the study of solutions of polynomial equations (algebraic geometry and number theory) and the study of symmetries of polynomials (invariant theory) were a major source of inspiration for the vast expansion of algebra in the 19th and 20th centuries.

In this course the algebra of polynomials in *n* variables over a field of coefficients is the basic object of study. The course covers fairly recent advances which have important applications to computer algebra and computational algebraic geometry (Gröbner bases - an extension of the Euclidean division algorithm to polynomials in 2 or more variables), together with a selection of more classical material.

Learning Outcomes

On successful completion of this course unit students will be able to demonstrate

- facility in dealing with polynomials (in one and more variables);
- understanding of some basic ideal structure of polynomial rings;
- appreciation of the subtleties of factorisation into prime and irreducible elements;
- ability to compute generating sets and Gröbner bases for ideals in polynomial rings;
- ability to relate polynomials to other algebraic structures (algebraic varieties and groups of symmetries);
- ability to solve problems relating to the factorisation of polynomials, irreducible polynomials and extension fields.

Future topics requiring this course unit

None, though the material connects usefully with algebraic geometry and Galois theory.

Syllabus

1. Factorisation: tests for irreducibility; Gauss' Lemma; Eisenstein's criterion; localisation and rings of fractions; Euclidean domains, principal ideal domains and unique factorisation domains. [6]
2. Ideals in commutative rings: Hilbert's basis theorem; noetherian rings; prime and maximal ideals. [4]
3. Gröbner bases: monomial orderings; division algorithm; Buchberger's algorithm; computing ideals in polynomial rings. [6]
4. Algebraically closed fields: adding roots of polynomials (review); algebraic and transcendental elements; algebraically closed fields. [2]

5. Zero sets of polynomials: affine varieties; radical of an ideal; Nullstellensatz (if time). [4]

Textbooks

The first two books are useful general references on algebra though neither covers Gröbner bases (which is a relatively new topic). For Gröbner bases see the book by Cox, Little and O'Shea. The last book is a new textbook which combines Gröbner bases with more traditional material.

- R.B.J.T. Allenby, *Rings, Fields and Groups: An Introduction to Abstract Algebra*, Edward Arnold, 0-3405-4440-6.
- J.B. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley,1994, 0-201-59291-6.
- D. Cox, J. Little and D. O'Shea, *Ideals, Varieties and Algorithms*, Springer 1992 (2nd edition) 1997, 0-387-94680-2.
- Niels Lauritzen, *Concrete Abstract Algebra*, Cambridge University Press 2004, 0-5215-3410-0.

Teaching and learning methods

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Course notes will be provided, as well as examples sheets and solutions. The notes will be concise and will need to be supplemented by your own notes taken in lectures, particularly of worked examples.

Assessment

Three pieces of coursework with the best 2 counting: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements