MANCHESTER 1824

MATH45111 - 2008/2009

General Information

- Title: Compressible and Incompressible Fluid Dynamics
- Unit code: MATH45111
- Credits: 15
- Prerequisites: MATH20502 *Fluid Mechanics*, MATH34001 *Applied Complex Analysis*.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. G. Vilenskiy

Specification

Aims

To demonstrate the power and beauty of modern methods used in aerodynamics to describe fluid flows past aerofoils, wings and other aerodynamic bodies both in subsonic and supersonic flows, and to enable students to gain physical insight into the nature of aerodynamic flows.

Brief Description of the unit

This course unit gives an introduction to fluid dynamics. This is a subdivision of continuum mechanics concerned with motion of liquids and gases under the condition that the moving matter is sufficiently dense to ignore the molecular structure of a medium and the motion of individual molecules. Fluid dynamics is aimed at predicting the velocity, pressure and temperature fields in the flows past rigid bodies. Of particular interest are the pressure and temperature distributions on the body surface. Being integrated they give the aerodynamic force acting upon the body and integral heat exchange between the flow and the body.

The main attention in this course unit will be with high Reynolds number flow, i.e. the flows with small viscosity. Most real flows observed in nature and encountered in engineering applications, in particular, in the aerospace industry, belong to this category. To describe such flows a variety of powerful mathematical techniques has been developed both for subsonic and supersonic flow regimes. These techniques and physical aspects of fluid motion will be the focus of attention in the course unit.

Learning Outcomes

Upon successful completion of the course, the students will

- be able to grasp the fundamental notions of fluid dynamics and operate with thermodynamic quantities of perfect gas;
- be able to derive and operate with equations of motion of inviscid fluid;
- be able to derive and operate with the integrals of incompressible and compressible fluid motion;
- know how to use complex potential to describe incompressible fluid motion past an aerofoil and, in particular, how to calculate the lift force;
- know how to use conservation laws to predict pressure, density and temperature distributions in compressible flows;
- know how to describe supersonic flows with shock waves;
- be able to use the Ackeret formula calculate pressure distribution over thin aerofoils in supersonic flows;
- be able to calculate the lift force using thin aerofoil theory in incompressible flows;
- know how to take account of compressibility effect in subsonic flows past thin aerofoils and wings.

Future topics requiring this course unit

None.

Syllabus

1. **Introduction.** Fundamentals of continuum mechanics. The continuum hypothesis. Knudsen number. Thermodynamic quasi-equilibrium. The notion of fluid particle. Kinematics of the flow field. Lagrangian description of the motion. Eulerian variables. Streamlines and pathlines. Vorticity and circulation.The continuity equation. Streamfunction and calculation of the mass flux. [4]

2. **Governing equations.** The Euler equation for inviscid compressible fluid motion. The energy and state equations. [3]
3. **Incompressible flows.** Integrals of motion. Kelvin's circulation theorem. Potential flows. Bernoulli's equation. Cauchy-Bernoulli integral for unsteady flows. Two-dimensional flows. Complex potential. Flow past circular cylinder. Conformal mapping. Joukovskii transformation. Flows past aerofoils. Lift and drag forces. Joukovskii-Kutta condition. [4]
4. **Compressible flows.** Integrals of motion. Bernoulli's equation and entropy conservation law. Crocco's integral. Potential flow equation. Theory of characteristics. Supersonic flows in physical and hodograph planes. Prandtl-Meyer expansion flow. Shock waves. [5]
5. **Thin aerofoils.** Supersonic flows. Ackeret formula. Evaluation of the lift and drag coefficients of aerofoils in supersonic flows. Incompressible flows. Thin aerofoil theory. Symmetrical and antisymmetrical solutions. Calculation of the lift force. Prandtl-Glauert rule. Calculation of the lift force and pressure distribution in subsonic flows past thin wings. [8]

Textbooks

- H. Lamb, *Hydrodynamics*, Dover, 6th edition 1993.
- G.K. Batchelor, *Introduction to Fluid Dynamics*, Cambridge 1973.
- H.L. Liepmann and A. Roshko, *Elements of Gasdynamics*, Wiley 1967.
- H. Glauert, *The Elements of Aerofoil and Airscrew Theory*, Cambridge 1948.

Teaching and learning methods

Three lectures and one examples class each week for weeks 1−5, 7−9 only.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements