MANCHESTER 1824

MATH42122 - 2008/2009

General Information

- Title: Galois Theory
- Unit code: MATH42122
- Credits: 15
- Prerequisites: MATH20212 *Algebraic Structures 2*, MATH32001/MATH42001 *Group Theory*.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. Ralph Stöhr

Specification

Aims

To introduce students to a sophisticated mathematical subject where elements of different branches of mathematics are brought together for the purpose of solving an important classical problem.

Brief Description of the unit

Galois theory is one of the most spectacular mathematical theories. It establishes a beautiful connection between the theory of polynomial equations and group theory. In fact, many fundamental notions of group theory originate in the work of Galois. For example, why are some groups called 'soluble'? Because they correspond to the equations which can be solved! (Solving here means there is a formula involving algebraic operations and extracting roots of various degrees that expresses the roots of the polynomial in terms of the coefficients.) Galois theory explains why we can solve quadratic, cubic and quartic equations, but no formulae exist for equations of degree greater than 4. It also gives a complete answer to ancient questions such as dividing a circle into n equal arcs using ruler and compasses. In modern language, Galois theory deals with 'field extensions', and the central topic is the 'Galois correspondence' between extensions and groups. Galois theory is a role model for mathematical theories dealing with 'solubility' of a wide range of problems.

Learning Outcomes

On successful completion of this course unit students will

- have deepened their knowledge about fields;
- have acquired sound understanding of the Galois correspondence between intermediate fields and subgroups of the Galois group;
- be able to compute the Galois correspondence in a number of simple examples;
- appreciate the insolubility of polynomial equations by radicals.

Future topics requiring this course unit

None.

Syllabus

1. Introduction and preliminaries: fields, vector spaces, homogeneous linear systems, polynomials. [4 lectures]
2. Field extensions, algebraic elements, Kronecker's construction. [4]
3. Splitting fields. [1]
4. Group characters, automorphisms and fixed fields. [2]
5. Normal extensions, separable polynomials, formal derivatives. [3]
6. The Fundamental Theorem of Galois Theory, Galois groups of polynomials, examples of the Galois correspondence. [3}
7. Finite fields, roots of unity, Noether's equations. [2]
8. Kummer extensions, [2]
9. Solutions of polynomial equations by radicals and an insolvable quintic. [2]

Textbooks

- E. Artin, *Galois Theory*, Dover Publications 1998.
- I Stewart, *Galois Theory*, 2nd edition, Chapman and Hall.
- J B Fraleigh, *A First Course in Abstract Algebra*, 5th edition, Addison-Wesley 1967.

Teaching and learning methods

2 lectures and one examples class each week. In addition students should expect to spend at least seven hours each week on private study for this course unit.

Assessment

Mid-semester coursework: one in-class test in the week after the Easter Vacation, weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements