MANCHESTER 1824

MATH31051/MATH41051 - 2008/2009

General Information

- Title: Introduction to Topology
- Unit code: MATH31051/MATH41051
- Credits: 10 (MATH31051), 15 (MATH41051)
- Prerequisites: MATH20122 *Metric Spaces* is desirable but not essential
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Peter Eccles

Specification

Aims

This lecture course aims to introduce students to the basic concepts of topological spaces and continuous functions, to give an introduction to combinatorial techniques in topology and to introduce homology theory.

Brief Description of the unit

This course unit is concerned with the study of topological spaces and their structure-preserving functions (continuous functions). Topological methods underpin a great deal of present day mathematics and theoretical physics. Topological spaces are sets which have sufficient structure so that the notion of continuity may be defined for functions between topological spaces. This structure is not defined in terms of a distance function but in terms of certain subsets known as open subsets which are required to satisfy certain basic properties. Continuous functions may stretch or bend a space and so two spaces are considered to be topologically equivalent if one can be obtained from the other by by stretching and bending: for this reason topology is sometimes called rubber sheet geometry.

The first half of the course unit introduces the basic definitions and standard examples of topological spaces as well as various types of topological spaces with good properties: path-connected spaces, compact spaces and Hausdorff spaces . The second half considers a particular class of topological spaces known as topological surfaces. To study these, basic ideas of combinatorial topology are introduced and these lead to the classification theorem for compact surfaces.

Learning Outcomes

On successful completion of this course unit students will be able to

- determine whether a collection of subsets of a set determines a topology;
- prove that two topological spaces are homeomorphic (topologically equivalent) or not homeomorphic;
- recognize whether or not a subset of a topological space is compact and be familiar with the basic properties of compact subsets and their proofs;
- recognize whether or not a topological space is Hausdorff and be familiar with the basic properties of Hausdorff spaces and their proofs;
- use the subspace topology, the product topology and the quotient topology;
- prove results about topological surfaces using cut and paste techniques;
- recognize when a collection of triangles in a Euclidean space forms a simplicial surface;
- recognize the underlying space of a simplicial surface by reducing a representing symbol to standard form;
- recognize the underlying space of a simplicial surface by calculating its Euler characteristic and determining whether it is orientable.

Future topics requiring this course unit

MATH31061/41061 *Calculus on Manifolds*, MATH31072 *Algebraic Topology*, MATH41101 *Geometric Cobordism Theory*

Syllabus

1. **Topological equivalence:** the topological equivalence of subsets of Euclidean spaces, path-connected sets and distinguishing subsets of Euclidean spaces using the cut point principle. [2 lectures]
2. **Topological spaces:** definition of a topology on a set, a topological space and a continuous function between topological spaces; closed subsets of a topological space; a basis for a topology. [3]
3. **Compactness:}**open coverings and subcoverings, definition of a compact subset of a topological space; basic properties of compact subsets; compact subsets in Euclidean spaces (the Heine-Borel Theorem). [2]
4. **Hausdorff spaces:** definition and basic properties of Hausdorff spaces. [1]
5. **Topological constructions:** subspaces, product spaces, quotient spaces; definitions and basic properties; standard examples including the Möbius band, the projective plane and the Klein bottle. [5]
6. **Topological surfaces:** definition and basic examples; the connected sum of two surfaces; the classification theorem for compact surfaces; handles and cross-caps. [2]
7. **Simplicial surfaces:** definition, a triangulation of a topological surface and the statement of the triangulation theorem for compact surfaces; representing the underlying space of a simplicial surface by a symbol; equivalent symbols and the statement and proof of the classification theorem for surface symbols; geometrical interpretation of the classification theorem. [3]
8. **Topological invariants of surfaces:** definition of the Euler characteristic and orientability of a simplicial surface; statement (no proof) of topological invariance; using these invariants to recognize the underlying space of a simplicial surface. [2]

For MATH41051 the lectures will be enhanced by additional reading on related topics.

Textbooks

The following book contains most of the material in the course.

- M. A. Armstrong. *Basic Topology.*

Teaching and learning methods

Three classes each week which will include opportunities to discuss problems from the Problems Sheets. In addition students should expect to spend at least four hours each week on private study for this course unit (seven hours for MATH41051).

Assessment

Mid-semester coursework: weighting 15% (MATH31051), 10% (MATH41051)
End of semester examination: two hours weighting 85% (MATH31051), three hours weighting 90% (MATH41051)

Arrangements