MATH45152 - 2008/2009

General Information

- Title: Inverse Problems and Imaging
- Unit code: MATH45152
- Credits: 15
- Prerequisites: First year core units.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr O. Dorn.

Specification

Aims

To introduce inverse and ill-posed problems and their application to industrial and medical imaging problems.

Brief Description of the unit

In science and engineering one often needs to infer the material properties of an object from some physical measurement. Typical examples are industrial and medical imaging: one applies for example ultra sound, X-rays or an electromagnetic field to an object, makes measurements on the outside and then attempts to form an image of the inside. These problems are typically ill-posed in the sense that the mapping taking data to image is discontinuous, and numerically reconstruction algorithms tend to be unstable unless one makes sufficient assumptions, such as smoothness, about the image. This course covers both the theory and practice of the process of reconstructing an image from measured data. The course will be illustrated by practical examples including visits to experimental groups in the University, and will include numerical examples illustrated by MATLAB programs.

Learning Outcomes

On successful completion of this course unit students will understand the basic theory of regularization for ill-posed problems, and its application to a number of medical and industrial problems.

Future topics requiring this course unit

None.

Syllabus

The course will include the following topics.

1. Introduction. Examples of well- and ill-posed problems, continuity. [2 lectures]
2. Discrete Linear Inverse Problems. Over and underdetermined systems, least squares problems, the Moore-Penrose generalized inverse, the singular value decomposition. [4]
3. Tikhonov regularization of linear ill-posed problems. [4]
4. Integral operators and ill-posed problems in Hilbert space. [4]
5. Iterative regularization methods, generalized Tikhonov regularization and the probabilistic interpretation of regularization, the discrete Picard condition and methods of choosing a regularization parameter. [4]
6. The Radon Transform and X-Ray computerized Tomography. [4]

Textbooks

- Beretro and Boccacci, *Introduction to Inverse Problems in Imaging*.
- Hansen, *Rank-deficient and Discrete Ill-posed Problems*.
- Aster, Borchers and Thurber, *Parameter Estimation and Inverse Problems*.
- Taraantola, *Inverse Problem Theory*,

- Natterer and Wubbeling, *Mathematical Methods in Image Reconstruction*.

Teaching and learning methods

Two lectures and an examples class each week with some supplementary reading. If only a few students wish to take the course unit it may be taught as a reading course.

Assessment

Coursework: case study of an applied problem weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements