



MATH45122 - 2008/2009

General Information

- Title: Non-Linear Waves
- Unit code: MATH45122
- Credits: 15
- **This course unit cannot be taken in the same semester as MATH35012 Waves.**
- Prerequisites: MATH20401 or MATH20411, MATH20502 *Fluid Mechanics*. MATH35012 *Waves* is useful but not essential.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. [Jitesh Gajjar](#)

Specification

Aims

The aim of the course is to introduce some ideas associated with nonlinear wave propagation such as wave steepening, shock formation, dispersion, and soliton properties, arising in simple physical systems.

Brief Description of the unit

Waves occur in many physical systems, and in the absence of nonlinear effects the appropriate wave equation can usually be solved analytically. Nonlinear effects bring new physics into play and shocks (jumps in solutions) can occur. The ideas used will be applied to explain, for example, bores on rivers and traffic flow problems. The breakdown of nonlinear wave systems will also be described. The methods used apply to a large variety of situations of practical importance.

Learning Outcomes

On successful completion of the course unit students will be able to

- understand the differences in the solution properties and physics of physical systems governed by the kinematic wave equation, the K - dV equation, Burger's equation, and the shallow water equations;
- solve $u_t + c(u)u_x = 0$ for given initial data and be able to identify the formation of shocks;
- solve the shallow water equations using the method of characteristics for simple flows;
- perform a phase plane analysis for the K - dV and related equations to identify travelling wave solutions, solitary wave solutions;
- use inverse scattering theory in simple situations.

Future topics requiring this course unit

None.

Syllabus

1. The hyperbolic wave $u_{tt} = c_0^2 D^2 u$, $u_t \pm c_0 u_x = 0$; wave forms; Fourier synthesis; dispersion; $C(k) = dw/dk$, group velocity; diffusion, e.g. Burger's linear equation $u_t + c_0 u_x = \nu u_{xx}$.
2. First order wave equation $u_t + c(u)u_x = 0$; characteristics; conservation ideas; conservation forms; traffic flow models. Waves in other physical systems.
3. Shallow water wave theory; the nonlinear equations; wave breaking, dam burst problems, via characteristics; linearisation and check against linear theory, and linear irrotational theory.
4. Irrotational water wave theory to obtain the Boussinesq equations; steady solutions of the Boussinesq equations; derivation of the Korteweg-de Vries equation from Boussinesq equations; conservation laws for K - dV; analytical solution of K - dV equation; the soliton; the inverse scattering method of solution.

5. Bäcklund transformations, Lax formulation.

Textbooks

- P.G. Drazin and R.S. Johnson, *Solitons, An Introduction*, CUP 1989.
- G.B. Whitham, *Linear and Non-linear Waves*, Wiley 1974.
- J. Stoker, *Water Waves*, Wiley Interscience 1957.
- L. Debnath, *Nonlinear Water Waves*, Academic Press 1994.

Teaching and learning methods

28 lectures and 8 examples classes.

Assessment

Mid-semester coursework: weighting 20%

End of semester examination: two and a half hours weighting 80%

Arrangements