MATH42042 - 2008/2009

General Information

- Title: Noncommutative Algebra
- Unit code: MATH42042
- Credits: 15
- Prerequisites: MATH32012 (Commutative Algebra) preferred
- Co-requisite units: None
- School responsible: Mathematics
- Member of staff responsible: Prof. Toby Stafford

Specification

Aims

To introduce students to noncommutative algebra.

Brief Description of the unit

Nature is inherently noncommutative---just try putting on your shoes and socks in the wrong order---and noncommutative structures are increasingly important throughout mathematics and physics. In this course, we will examine in detail some of the most important noncommutative algebras that appear "in nature" and prove some of the basic structure theorems about noncommutative rings. One of the most fundamental such algebras is the quaternions---if you ignore the fact that it is not commutative, then this is a field that is 4-dimensional as a real vector space! Famously when Hamilton discovered it in the 19-th century he carved its formulae on a bridge lest he forget them. Another is the Weyl algebra---sometimes called the "algebra of quantum mechanics" since its structure encodes the Uncertainty Principle, and sometimes called a ring of differential operators since it encodes the algebraic aspects of differential equations.

Learning Outcomes

On successful completion of this course unit students will

- have a deepened understanding of algebra;
- understand fundamental noncommutative algebras like the quaternions and the Weyl algebra (or algebra of quantum mechanics);
- understand some of the basic structure theorems in noncommutative algebra, like the Wedderburn density theorem, the classification of simple Artinian rings and the structure of the Jacobson radical.

Future topics requiring this course unit

None

Syllabus

1. Introduction and preliminaries: fields, rings and matrices;
2. The quaternions;
3. Wedderburn-Artin theory---many rings are matrices;
4. Jacobson radical theory;
5. Introduction to representation theory;
6. The Weyl algebra or algebra of quantum mechanics;
7. Division rings.

Textbooks

- Cohn, P. M. Introduction to ring theory. Springer-Verlag, 2000.
- Passman, D. S. A course in ring theory. The Wadsworth & Brooks/Cole Mathematics Series, 1991.

- Lam, T. Y. A first course in noncommutative rings. Second edition. Graduate Texts in Mathematics, 131. Springer-Verlag, New York, 2001

Teaching and learning methods

3 lectures each week with an additional office hour for informal discussion. In addition students should expect to spend at least seven hours each week on private study for this course unit.

Assessment

Coursework: weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements