MANCHESTER 1824

MATH46111/MATH66111 - 2008/2009

General Information

- Title: Numerical Functional Analysis
- Unit code: MATH46111/MATH66111
- Credits: 15
- Prerequisites:
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. T. Betcke

Specification

Aims

To develop understanding of functional analysis. To establish existence and uniqueness of solutions for a number of important mathematical models. To derive numerical algorithms for PDEs and prove rigorous error estimates. To understand integral equations and their numerical properties.

Brief Description of the unit

This course unit introduces fundamental tools from functional analysis and uses them to develop a theory of existence and uniqueness for some important PDEs from mathematical physics with an emphasis on numerical approximations. We start with fundamental concepts from functional analysis involving Banach and Hilbert spaces, compactness and linear operators. We then apply these tools to develop the right framework to show existence and uniqueness of some important PDEs. Galerkin methods are discussed to approximate them. We go on to some advanced topics involving Fourier transforms in Lebeque spaces, spectral theory and integral equations of the first and second kind. A strong emphasis throughout the whole course is put on the numerical implementation of the discussed techniques.

Learning Outcomes

On successful completion of this course unit students will

- understand the concept of Banach and Hilbert spaces and some of their theory;
- understand the concepts of weak derivatives and Sobolov spaces
- understand the importance of existence and uniqueness theory and how it is developed for the Poisson equation, the heat equation and the Helmholtz equation.
- be able to understand the finite difference and finite element approximations and prove error estimates
- understand fundamental results about the spectral theory of linear operators
- understand examples of integral equations of the first and second kind and their different numerical behaviour

Future topics requiring this course unit

None.

Syllabus

1. **Introduction.** Well posedness. Review of Cauchy sequences. Banach and Hilbert spaces. [2 lectures]
2. **Tools from functional analysis.** Fundamental results about Banach and Hilbert spaces, linear operators, symmetric and compact operators [3 lectures]
3. **Fourier Series and Fourier transformations.** Complete orthonormal bases. Definition of Fourier series and Fourier transformations. Plancherel's theorem. [2 lectures]
4. **Model Diffusion Problem.** Sobolev spaces and generalised derivatives. Classical and weak solutions of model diffusion problem. Existence and uniqueness of solution. Riesz Representation Theorem. [6 lectures]
5. **Galerkin Approximation and the Finite Element Method.** Classical and weak solutions. Galerkin approximation and convergence. Finite element spaces. [5 lectures]

6. **Spectral Theory of Linear Operators.** Eigenfunctions and complete orthonormal basis. Hilbert Schmidt Theorem. Unbounded operators. Laplace eigenvalue problems [4 lectures]
7. **Linear Integral Equations.** Integral equations of the first and second kind, Fredholm theory, numerical approximation and conditioning of integral equations, applications to Helmholtz problems [5 lectures]

Textbooks

- Howard Elman, David Silvester and Andy Wathen, *Finite Elements and Fast Iterative Solvers*, Oxford University Press, Oxford 2005, ISBN 0-19-852868-X (paperback).
- Brenner, Scott, *The Mathematical Theory of Finite Element Methods - 2nd Edition*, Springer, 2002, ISBN 0-387-95451-1
- Renardy and Rogers, *An Introduction to Partial Differential Equations*, Springer 1993
- Lieb and Loss, *Analysis* AMS, 2001 ISBN 0-8218-2783-9
- E. B. Davies, *Spectral Theory and Differential Operators* Cambridge University Press, Cambridge 1995, ISBN 0-521-47250-4
- Kress, *Linear Integral Equations - 2nd edition*, Springer, 1999, ISBN 0-387-98700-2

Teaching and learning methods

27 lectures (two or three lectures per week) with a weekly examples class. In addition should expect to spend at least seven hours each week on private study.

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements