MATH43001 - 2008/2009

General Information

- Title: Predicate Calculus
- Unit code: MATH43001
- Credits: 15
- Prerequisites: MATH20302 *Propositional Logic*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Markus Tressl

Specification

Aims

To introduce students to the formal notions of language, proof, semantics, and completeness with quantificational logic, in order to:

- improve their understanding and appreciation of the foundations of mathematics and
- provide the necessary background knowledge for later logic course units.

Brief Description of the unit

What do we mean by saying that a mathematical statement is true? What exactly is a mathematical proof? Can mathematics be reduced to computer programming? These are some of the questions which motivate the study of mathematical logic. Predicate logic provides remarkable insight into these questions by providing a precise formalism capable of expressing all ordinary mathematics. The module will lead up to a proof of the completeness theorem, a striking result of Kurt Gödel (1930), which demonstrates the equivalence of a natural notion of logical consequence with provability in a certain axiomatic system.

Learning Outcomes

On successful completion of this course unit students will

- appreciate how arguments involving predicates can be formalised semantically and syntactically and how these are connected (via the Completeness Theorem);
- in simple cases be able to show that 'A follows from B' both by giving a semantic argument and by constructing a formal proof;
- in simple cases be able to show that 'A does not follow from B' by using semantics.

Future topics requiring this course unit

MATH43042 *Gödel's Theorems*, MATH43052 *Model Theory*

Syllabus

1. **Introduction.** Review of propositional logic. Motivation for the study of predicate logic with examples of reasoning with quantifiers. [2 lectures]
2. **Truth.** Languages for predicate logic. Signatures and structures. Formulae, sentences and Tarski's definition of Truth. Logical consequence, logical equivalence and logical validity. Theories and models. [9]
3. **Proof.** An axiom system for predicate logic. The Soundness Theorem. Consistency. [6]
4. **Completeness**. The completeness theorem for predicate logic. Simple applications. [7]
   Models. An introduction to model theory. The compactness and Lowenheim-Skolem theorem with applications. [6]

Textbooks

- H.B. Enderton, *A Mathematical Introduction to Logic*, Academic Press.

- D. van Dalen, *Logic and Structure*, Springer-Verlag, 3rd edition 1997.
- A.G. Hamilton, *Logic for Mathematicians*, Cambridge University Press, revised edition 1988.

Teaching and learning methods

30 lectures, 6 examples classes, and assigned reading.

Assessment

Mid-semester coursework: two take home tests weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements

Reading Assignments

Some of the details concerning languages for predicate logic with function symbols and equality and the proof of the completeness theorem will not be covered in lectures but left as reading assignments.