MATH43021 - 2008/2009

General Information

- Title: Set Theory
- Unit code: MATH43021
- Credits: 15
- Prerequisites: MATH20302 *Propositional Logic*.
- Co-requisite units: Some familiarity with predicate logic, as studied in the course unit MATH43001 would be helpful, but is not essential.
- School responsible: Mathematics
- Members of staff responsible: Prof. Peter Aczel

Specification

Aims

To introduce students to the elements of set theory and its role as a foundation for mathematics.

Brief Description of the unit

The notion of set is one of the fundamental notions of modern mathematics. Set Theory was initiated by Cantor over a hundred years ago. He developed a revolutionary theory of transfinite numbers that can be used to compare the `sizes' of possibly infinite sets. A naive approach to set theory leads to paradox and Zermelo initiated an axiomatic approach that puts set theory on a sound rigorous basis. Axiomatic set theory can be viewed as a foundation of mathematics in the following sense. All mathematical notions can be defined in purely set theoretical terms and their properties can be proved using only the set theoretical axioms. Also the language of set theory has played a central unifying role in modern mathematics.

The course unit will present Cantor's theory of transfinite numbers, the axiom system ZF and the axiom of choice.

Learning Outcomes

On successful completion of this course unit students will have acquired

- facility with the notions of elementary set theory;
- a sound knowledge of the basic properties of the cardinal and ordinal numbers;
- familiarity with the axiom system ZF and its role as a foundation for mathematics,
- an understanding of the axiom of choice and some of its applications.

Future topics requiring this course unit

None.

Syllabus

Part I (3 lectures)

1. Review of elementary set theory
2. Naive and not so naive set theory

Part II (15 lectures)

3. Finite and Countable sets
4. Cardinal numbers and their ordering
5. The arithmetic of cardinal numbers
6. The comparison of cardinal numbers

Part III (12 lectures)

7. Well-orderings and initial ordinals
8. The arithmetization of analysis
9. The Zermelo-Fraenkel set theory ZFC
10. The von-Neumann ordinals

Textbooks

There is no recommended textbook to cover the course. The following are some good books to consult.

- A. G. Hamilton, *Numbers, Sets and Axioms*, CUP Press (in paperback).
- Y. N. Moschovakis, *Notes on Set Theory*, Springer-Verlag Undergraduate Texts in Mathematics.
- H. B. Enderton, *Elements of Set Theory*, Academic Press.

Teaching and learning methods

30 lectures, 6 examples classes. In addition students are expected to do at least seven hours a week of private study on this course unit.

Assessment

Coursework: two take home tests; weighting 20%
End of semester examination: two and a half hours; weighting 80%

Arrangements