MANCHESTER 1824

MATH35001 - 2008/2009

General Information

- Title: Viscous Fluid Flow
- Unit code: MATH35001
- Credits: 10
- Prerequisites: MATH20401 or MATH20411. MATH20502 *Fluid Mechanics* is useful but not essential.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. Matthias Heil

Specification

Aims

The course will provide an introduction to the mathematical theory of viscous fluid flows. After deriving the governing equations from a general continuum mechanical approach, the theory will be applied to a variety of practically important problems.

Brief Description of the unit

This course is concerned with the mathematical theory of viscous fluid flows. Fluid mechanics is one of the major areas for the application of mathematics and has obvious practical applications in many important disciplines (aeronautics, meteorology, geophysical fluid mechanics, biofluid mechanics, and many others). Using a general continuum mechanical approach, we will first derive the governing equations (the famous Navier-Stokes equations) from first principles. We will then apply these equations to a variety of practical problems and examine appropriate simplifications and solution strategies.

Many members of staff in the School have research interests in fluid mechanics and this course will lay the foundations for possible future postgraduate work in this discipline.

Learning Outcomes

On successful completion of this course unit students will be able to

- understand the continuum mechanical derivation of the Navier-Stokes equations and the appropriate boundary conditions;
- understand the kinematics of fluid flow;
- apply the equations to various fluid problems giving a mathematical description of the flow, and to solve some of these problems.

Future topics requiring this course unit

MATH45132 *Hydrodynamic Stability Theory.*

Syllabus

1. Introduction; overview of the course; introduction to index notation. [2 lectures]
2. The kinematics of fluid flow: The Eulerian velocity field; the rate of strain tensor and the vorticity vector; the equation of continuity. [3]
3. The Navier-Stokes equations: The substantial derivative; the stress tensor; Cauchy's equation; the constitutive equations for a Newtonian fluid. [4]
4. Boundary and initial conditions; surface traction and the conditions at a free surface. [1]
5. One-dimensional flows: Couette/Poiseuille flow; flow down an inclined plane; the vibrating plate. [3]
6. The equations in curvilinear coordinates; Hagen-Poiseuille flow; circular Couette flow. [2]
7. Dimensional analysis and scaling; the dimensionless Navier-Stokes equations and the importance of the Reynolds number; limiting cases and their physical meaning; lubrication theory. [3]
8. The stream function/vorticity equations. [2]
9. Stokes flow (zero Reynolds number flow). [2]

10. High-Reynolds number flow; boundary layers; the Blasius boundary layer. [2]

Textbook

- Spiegel, M., *Vector Calculus*, McGraw Hill (Schaum's Outline series) 1974.
- Batchelor, G.K., *An Introduction to Fluid Dynamics*, Cambridge 1967.
- Sherman, F.S., *Viscous Flow*, McGraw Hill 1990.
- McCormack , P.S. and Crane, L.J.,*Physical Fluid Dynamics*, Academic Press 1973.
- Panton, R.L., *Incompressible Flow*, (second edition), Wiley 1996.
- White, F.M., *Viscous Fluid Flow*, (second edition), McGraw Hill 1991.

Teaching and learning methods

Two lectures and an examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Assessment

Weekly coursework, worth 20% of the final mark, throughout the semester.
End of semester examination: two hours weighting 80%.

Arrangements