

MATH20132 - 2009/2010

General Information

- Title: Calculus of Several Variables
- Unit code: MATH20132
- Credit rating: 10
- Level: 2
- Pre-requisite units: MATH10121 or MATH10131, MATH10202 or MATH10212, MATH201101 or MATH20111
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Prof. Peter J. Eccles

Unit specification

Aims

Brief description

Functions of several variables were briefly considered in first year calculus. Although there are some similarities with the familiar theory of one real variable, the theory for functions of several variables is far richer. For example, for functions of several variables, the critical points might be maxima, minima or saddle points (which are minima in one direction and are maxima in another direction). A key idea is to generalize the definition of the derivative to the case of maps $f: R^n \rightarrow R^m$. This is the Fréchet derivative, which is a linear map $Df: R^n \rightarrow R^m$ (often represented by a matrix) which gives the best approximation to the function. This course deals with a number of very elegant and useful results, including the Inverse Function Theorem, the Implicit Function Theorem and a study of the critical points of functions of several variables.

Intended learning outcomes

Future topics requiring this course unit

The ideas in this course are used in many areas of pure and applied mathematics. A natural follow on course unit is the level 3 *Calculus on Manifolds*.

Syllabus

1. The differential of a map $f: R^n \rightarrow R^m$.
2. An example-application: hydrodynamic interpretation of a vector field on Euclidean space; decomposition of its differential into symmetric ('deformation') and antisymmetric ('rotation') parts, and the geometric meaning of divergence and curl.
3. Differentials and partial derivatives; Taylor series.
4. Chain rule; Inverse and Implicit Function Theorems.
5. Critical points and values.

6. Saard's Theorem and its applications.
7. Study of critical points; the Hessian matrix.
8. Conditional extremum and Lagrange multipliers.

Textbooks

M.J. Field, *Differential Calculus and its Applications*, Van Nostrand 1976.

W. Fleming, *Functions of Several Variables*, Addison-Wesley 1965.

J. and B. Hubbard, *Vector Calculus, Linear Algebra, and Differential Forms*, Prentice Hall 1998.

Learning and teaching processes

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Assessment

Coursework; Homework, set in Week 8, deadline in Week 9, Weighting within unit 15%

2 hours end of semester examination; Weighting within unit 85%

Arrangements
