MATH20142 - 2009/2010

General Information

- Title: Complex Analysis
- Unit code: MATH20142
- Credit rating: 10
- **This course unit cannot be taken by students who have taken MATH20101 *Real and Complex Analysis*.**
- Level: 2
- Pre-requisite units: MATH10111, MATH10131, MATH20111
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Nikita Sidorov

Unit specification

Aims

The programme unit aims to introduce the basic ideas of complex analysis, with particular emphasis on Cauchy's Theorem a nd the calculus of residues.

Brief description

This course introduces the calculus of complex functions of a complex variable. It turns out that complex differentiability is a very strong condition and differentiable functions behave very well. Integration is along paths in the complex plane. The central result of this spectacularly beautiful part of mathematics is Cauchy's Theorem guaranteeing that certain integrals along closed paths are zero. This striking result leads to useful techniques for evaluating real integrals based on the `calculus of residues'.

Intended learning outcomes

On completion of this unit successful students will be able to:

- understand the significance of differentiability for complex functions and be familiar with the Cauchy-Riemann equations;
- evaluate integrals along a path in the complex plane and understand the statement of Cauchy's Theorem;
- compute the Taylor and Laurent expansions of simple functions, determining the nature of the singularities and calculating residues;
- use the Cauchy Residue Theorem to evaluate integrals and sum series.

Future topics requiring this course unit

Complex analysis is needed for advanced analysis, geometry and topology, but also has applications in differential equations, potential theory, fluid mechanics, asymptotics and wave analysis.

Syllabus

1. **Series.** Complex series, power series and the radius of convergence. [2 lectures]
2. **Continuity.** Continuity of complex functions [2]
3. **The complex plane.** The topology of the complex plane, open sets, paths and continuous functions. [2]
4. **Differentiation.** Differentiable complex functions and the Cauchy-Riemann equations. [2]
5. **Integration.** Integration along paths, the Fundamental Theorem of Calculus, the Estimation Lemma, statement of Cauchy's Theorem. [4]
6. **Argument and Logarithm.** [2]
7. **Taylor and Laurent Series.** Cauchy's Integral Formula and Taylor Series, Zeros and Poles, Laurent Series. [3]
8. **Residues.** Cauchy's Residue Theorem, the evaluation of definite integrals and summation of series. [5]

Textbooks

- Ian Stewart and David Tall, *Complex Analysis*, Cambridge University Press, 1983.

Learning and teaching processes

Two lectures and one examples class each week. In addition students should expect to do at last four hours private study each week for this course unit.

Assessment

Coursework; Weighting within unit 20%
2 hours end of semester examination; Weighting within unit 80%

Arrangements