This is archived information. Please visit http://maths.manchester.ac.uk for current course unit information.

MATH32052/MATH42052 - 2009/2010

General Information

- Title: Hyperbolic Geometry
- Unit code: MATH32052/42052
- Credits: 10 (MATH32052), 15 (MATH42052)
- Prerequisites: Good Knowledge of Complex numbers.
- Co-requisite units: None
- School responsible: Mathematics
- Member of staff responsible: Dr. Charles Walkden

Specification

Aims

To provide an introduction to the hyperbolic plane and hyperbolic geometry. To study how discrete groups of isometries act on the hyperbolic plane.

Brief Description of the unit

Consider the Euclidean plane R^2. If we take a straight line L and a point p not on that line, then there is a unique straight line through p that never intersects L (draw a picture!). This is Euclid's parallel postulate. Euclid introduced several axioms for what is now called Euclidean geometry (that is, geometry in R^2 or more generally in R^n and a great deal of effort was employed in attempting to prove that these axioms implied the parallel postulate. However, in early 19th century, the hyperbolic plane was introduced as a setting in which Euclid's axioms hold but the parallel postulate fails: there may be infinitely many "straight" lines through a point that do not intersect a given "straight" line.

Today, hyperbolic geometry is a rich and active area of mathematics with many beautiful theorems (and can be used to generate very attractive pictures)

This course provides an introduction to hyperbolic geometry. We start by discussing what is meant by "distance" and what is "straight" about a straight line in the Euclidean plane R^2. We then give an introduction to the hyperbolic plane. Topics include: distance and area in the hyperbolic plane, distance-preserving maps, hyperbolic trigonometry and hyperbolic polygons.

The collection of all distance-preserving maps forms a group. The second part of the course studies a particular class of such groups, namely Fuchsian groups. By using a very beautiful theorem called Poincaré's Theorem, we will describe the connections between such groups and tessellations (tilings) of the hyperbolic plane. The emphasis here will be on how to calculate with and apply Poincaré's Theorem, rather than on rigorous proofs.

One aim of the course is to show how results and techniques from different areas of mathematics, notably geometry, algebra and analysis, can be used coherently in the study of a single topic.

Learning Outcomes

On successfully completing the course students will be able to:

- understand the geometry of the hyperbolic plane and be able to prove results and identities in hyperbolic geometry and trigonometry,
- understand other models of the hyperbolic plane, such as the Poincaré disc model,
- understand how the group of Möbius transformations acts on the hyperbolic plane by isometries,
- understand how discrete groups of isometries act on the hyperbolic plane and their connections with fundamental domains and Dirichlet regions.

Future topics requiring this course unit

None

Syllabus

- Introduction, background and motivation.
- The upper half-plane model, hyperbolic distance and area, geodesics. The group of Möbius transformations as isometries.
- The Poincaré disc model. Möbius transformations of the Poincaré disc.
- Hyperbolic triangles, hyperbolic trigonometry, hyperbolic polygons
- Classifying different types of isometries.
- Introduction to discrete groups of isometries.
- Fundamental domains and Dirichlet regions.
- Poincaré's theorem and groups generated by side-pairing transformations.

Textbooks

J. Anderson, *Hyperbolic Geometry*, Springer, 1999.
S. Katok, *Fuchsian Groups,* Chicago, 1992
A. Beardon, *The Geometry of Discrete Groups*, Springer, 1983

The book by Anderson is the most suitable for the course.

Teaching and learning methods

Two lectures a week and a weekly examples class. In addition students should expect to do at least four hours private study each week for this course unit (and seven hours for MATH42052).

Assessment

MATH32052: Two hours end of semester examination; Weighting within unit 100%
MATH42052: Three hours end of semester examination; Weighting within unit 100%

Arrangements