MANCHESTER 1824

MATH10212 - 2009/2010

General Information

- Title: Linear Algebra
- Unit code: MATH10212
- Credits: 15
- Prerequisites: A-Level Mathematics or equivalent
- Co-requisite units: This course unit can only be taken with MATH10232 *Calculus and Applications* or MATH10111 *Sets, Numbers and Functions*.
- School responsible: Mathematics
- Members of staff responsible: Prof. A. Borovik

Specification

Aims

This course unit aims to introduce the basic ideas and techniques of linear algebra for use in many other lecture courses. The course will also introduce some basic ideas of abstract algebra and techniques of proof which will be useful for future courses in pure mathematics.

Brief Description of the unit

This core course aims at introducing students to the fundamental concepts of linear algebra culminating in abstract vector spaces and linear transformations. The first part covers systems of linear equations, matrices, and some basic concepts of the theory of vector spaces in the concrete setting of real linear n-space, $\boldsymbol{R}^n$. The second part briefly explores orthogonality, and then goes on to introduce abstract vector spaces over arbitrary fields and linear transformations. The subject material is of vital importance in all fields of mathematics and in science in general.

Learning Outcomes

On successful completion of this course unit students will be able to

- be able to solve systems of linear equations by using Gaussian elimination to reduce the augmented matrix to row echelon form or to reduced row echelon form;
- understand the basic ideas of vector algebra: linear dependence and independence and spanning;
- be able to apply the basic techniques of matrix algebra, including finding the inverse of an invertible matrix using Gauss-Jordan elimination;
- know how to find the row space, column space and null space of a matrix, to find bases for these subspaces and be familiar with the concepts of dimension of a subspace and the rank and nullity of a matrix, and to understand the relationship of these concepts to associated systems of linear equations;
- be able to find the eigenvalues and eigenvectors of a square matrix using the characteristic polynomial and will know how to diagonalize a matrix when this is possible;
- be able to find the orthogonal complement of a subspace;
- be able to recognize and invert orthogonal matrices;
- be able to orthogonally diagonalize symmetric matrices;
- be familiar with the general notions of a vector space over a field and of a subspace, linear independence, dependence, spanning sets, basis and dimension of a general subspace;
- be able to find the change-of-basis matrix with respect to two bases of a vector space;
- be familiar with the notion of a linear transformation, its matrix with respect to bases of the domain and the codomain, its range and kernel, and its rank and nullity and the relationship between them;
- be familiar with the notion of a linear operator and be able to find the eigenvalues and eigenvectors of an operator.

Future topics requiring this course unit

Almost all Mathematics course units, particularly those in pure mathematics.

Syllabus

Systems of linear equations: coefficient matrix and augmented matrix, elementary row operations, row echelon form and reduced row echelon form, solution of systems using Gauss and Gauss-Jordan algorithms. [Poole 2.1, 2.2, 2.3; 4 Lectures]

Algebra of matrices: addition, multiplication, inverse, transpose, elementary matrices and elementary row operations, row equivalence, calculation of inverses by Gauss-Jordan algorithm, finding non singular $\boldsymbol{P}$ so that $\boldsymbol{PA}$ is in (reduced) row echelon form, row canonical form. [Poole 3.1, 3.2, 3.3, 3.4, 3.7; 6 Lectures]

Subspaces of $\boldsymbol{R}^n$: the null space and column space of a matrix $\boldsymbol{A}$ and their significance for $\boldsymbol{Ax = b}$, linear independence, spanning sets, basis of a subspace, dimension of a subspace, using Gauss algorithm to find a basis for null space, column space and row space, nullity and rank of an $m \times n$ matrix, *nullity + rank = n*, *row rank = column rank*, coordinates and change of coordinates. [Poole 3.5; 6 Lectures]

Standard inner product on $\boldsymbol{R}^n$: standard inner product, length of vector, orthogonal vectors, orthonormal basis, coordinates with respect to an orthonormal basis, null space of $\boldsymbol{A}$ is orthogonal complement of row space. [Poole 5.1, 5.2, 5.3; 3 Lectures]

Vector spaces over a field $\boldsymbol{R}$: definition and examples (including function spaces, spaces of matrices), basis and dimension, basis, dimension, coordinates, change of coordinates, subspaces, intersections, sum and direct sum, formula for $dim(U + V)$. [Poole 6.1, 6.2, 6.3; 4 Lectures]

Linear transformations: examples, monomorphisms, epimorphisms, isomorphisms, basis gives isomorphism with $\boldsymbol{R}^n$, kernel, image, nullity, rank, cosets of kernel and solutions of $f(x) = b$, application to linear equations, linear constant coefficient differential equations, matrix of a linear map with respect to bases, change of bases. [Poole 6.4, 6.5, 6.6; 4 Lectures]

Determinants: properties and methods of calculation (without proofs). [Poole 4.2; 2 Lectures]

Linear operators: change of coordinates and similarity of matrices, the diagonalization problem, eigenvectors and eigenvalues, characteristic polynomial, diagonalization of $n \times n$ matrices with n distinct eigenvalues. [Poole 6.6, 4.3, 4.4, 4.6; 4 Lectures]

Remarks. Most examples will be over the field of real numbers $\boldsymbol{R}$ but the theory works over any field, in particular $\boldsymbol{C}$ and $\boldsymbol{F}_p$.

Textbooks

The course is based on the course text which students are expected to buy:

- David Poole, *Linear Algebra: a Modern Introduction, Thomson, second edition 2006*, International Student Edition: ISBN 0-534-40596-7.

There is a useful website associated with the text.

Notes will be issued for the material not covered in the course text.

Teaching and learning methods

3 lectures and 1 example class per week

Assessment

Supervision attendance and participation; Weighting within unit 10%
Coursework; Weighting within unit 15%
Two and a half hours end of semester examination; Weighting within unit 75%

Arrangements