MATH31001/MATH41001 - 2009/2010

General Information

- Title: Linear Analysis
- Unit code: MATH31001/MATH41001
- Credits: 10 (MATH31001), 15 (MATH41001)
- Prerequisites: MATH20101 or MATH20111, MATH20122 *Metric Spaces*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. R. Sharp

Specification

Aims

To give an introduction to Modern Analysis, including elements of functional analysis. The emphasis will be on ideas and results.

Brief Description of the unit

This course unit deals with a coherent and elegant collection of results in analysis. The aim of this course is to provide an introduction to the theory of infinite dimensional linear spaces, which is not only an important tool, but is also a central topic in modern mathematics.

This area has many applications to other areas in Pure and Applied Mathematics such as Dynamical Systems, C* algebras, Quantum Physics, Numerical Analysis, etc.

Learning Outcomes

On successful completion of this module students will be able to

- understand the approximation of continuous functions using the Stone-Weierstrass Theorem;
- understand the concepts of Hilbert and Banach spaces, with l^2 and l^p spaces serving as examples;
- understand the definitions of linear functionals and dual spaces, prove the Riesz representation theorem for Hilbert spaces;
- define linear operators, their spectrum (with matrices serving as examples) and their spectral radius, understand the definitions of self-adjoint, isometric and unitary operators on Hilbert spaces and their spectra, be able to apply these ideas to matrices.

Students will have seen proofs of the main results, although an intimate knowledge of the full details is not required.

Future topics requiring this course unit

This course would be useful to students interested in the following topics: MATH31011/MATH41011 *Measure and Fractals* MATH41112 *Ergodic Theory*.

Syllabus

1. Revision of compactness and uniform continuity. [2 lectures]
2. Approximation by polynomials and the Stone-Weierstrass Theorem. [4]
3. Normed vector spaces. Finite and infinite dimensional spaces. Completeness and Banach spaces. [4]
4. Sequence spaces. Continuous function spaces. Separability. Hilbert spaces, orthogonal complements and direct sum decompositions, orthonormal bases. [6]
5. Bounded linear functionals. The Hahn-Banach theorem. Dual spaces. The Riesz representation theorem for Hilbert spaces. [4]
6. Bounded linear operators and their norms. Compact operators. Invertible operators. Spectra. [4]
7. Linear operators on Hilbert spaces. Self-adjoint and unitary operators and their spectra. [4]

For MATH41001, the lectures will be enhanced by additional reading on the proof of the Stone-Weierstrass Theorem, l^p spaces and the proof of the Hahn-Banach Theorem.

Textbooks

- G. F. Simmons, *An Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
- I.J. Maddox, *Elements of Functional Analysis*, C.U.P. 1989.
- B.P. Rynne and M.A. Youngson, Linear Functional Analysis, Springer S.U.M.S., 2008.

Teaching and learning methods

Two lectures and an examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit (seven hours for MATH41001).

Assessment

End of semester examination: two hours weighting 100% (MATH31001), three hours weighting 100% (MATH41001)

Arrangements