



MATH20602 - 2009/2010

General Information

- Title: Numerical Analysis I
- Unit code: MATH20602
- Credit rating: 10
- Level: 2
- Pre-requisite units: MATH20401 or MATH20411
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Dr. Ruth Thomas

Unit specification

Aims

The programme unit aims to introduce students to theoretical and practical aspects of the numerical solution of linear and nonlinear equations, the approximation of functions by polynomials and the approximation of integrals via quadrature schemes.

Brief description

Numerical analysis is concerned with finding numerical solutions to problems for which analytical solutions either do not exist or are not readily or cheaply obtainable. This course provides an introduction to the subject, focusing on the three core topics of iteration, interpolation and quadrature.

The module starts with 'interpolation schemes', methods for approximating functions by polynomials, and 'quadrature schemes', numerical methods for approximating integrals, will then be explored in turn. The second half of the module looks at solving systems of linear and nonlinear equations via iterative techniques. In the case of linear systems, examples will be drawn from the numerical solution of differential equations.

Students will learn about practical and theoretical aspects of all the algorithms. Insight into the algorithms will be given through MATLAB illustrations, but the course does not require any programming.

Intended learning outcomes

On completion of this unit successful students will be able to:

- practical knowledge of a range of iterative techniques for solving linear and nonlinear systems of equations, theoretical knowledge of their convergence properties, an appreciation of how small changes in the data affect the solutions and experience with key examples arising in the solution of differential equations;
- practical knowledge of polynomial interpolation, its numerical implementation and theoretical knowledge of associated approximation properties;

- practical knowledge of quadrature schemes and theoretical knowledge of their associated approximation properties.

Future topics requiring this course unit

MATH36022 Numerical Analysis 2, MATH46101 Numerical Linear Algebra, MATH46142 Finite Element Analysis.

Syllabus

1. **Introduction to numerical analysis.** Floating point arithmetic. Catastrophic cancellation and the quadratic equation formula. Efficiency and Horner's method. [2 lectures]
2. **Approximation.** Lagrange interpolation. Uniqueness and existence of interpolants. Error estimates. Runge's example. Divided difference form of interpolant. Application to quadrature. [6]
3. **Linear Algebra.** PDE example to introduce sparse matrices. Iterative vs direct methods. Examples of iterative methods (Jacobi, Gauss-Seidel). Vector Norms. Eigenvalues, eigenvectors, spectral radius. Convergence criteria. Error bounds, matrix norms, and condition number. [7]
4. **Solving nonlinear equations.** Solution of nonlinear equations by the bisection method, fixed point iteration, and Newton's method. Discussion in one and two dimensions. [6]

Textbooks

- Endre Suli and David Mayers, *An Introduction to Numerical Analysis*, Cambridge University Press 2003.
- Richard L. Burden and J. Douglas Faires, *Numerical Analysis*, Brooks Cole 2004.
- Desmond J. Higham and Nicholas J. Higham, *MATLAB Guide*, Second edition, SIAM 2005.

Learning and teaching processes

Two lectures and one examples class each week. In addition students should expect to do at least four hours work each week on this course unit.

Assessment

Coursework; Weighting within unit 20%

2 hours end of semester examination; Weighting within unit 80%

Arrangements
