MATH46111/MATH66111 - 2009/2010

General Information

- Title: Numerical Functional Analysis
- Unit code: MATH46111/MATH66111
- Credits: 15
- Prerequisites:
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. T. Shardlow

Specification

Aims

To develop understanding of functional analysis. To establish existence and uniqueness of solutions for a number of important mathematical models. To derive numerical algorithms for PDEs and prove rigorous error estimates. To understand integral equations and their numerical properties.

Brief Description of the unit

This module develops the rigorous analysis of PDEs with special emphasis on numerical approximation. After developing a number of tools in functional analysis, we consider the well posedness of some PDEs of mathematical physics: we formulate precise definitions of solution and show these solutions exists and are unique. We go on to develop numerical algorithms and prove rigorous estimates in simple caes. We look at finite differences for ODEs, the Galerkin approximation for the Poisson equation, and a spectral/finite difference approximation to the heat equation. We conclude with an introduction to stochastic PDEs, which is currently a very active research area.

Learning Outcomes

On successful completion of this course unit students will

- understand the concept of Banach and Hilbert spaces and some of their theory;
- understand the concepts of weak derivatives and Sobolev spaces
- understand the importance of existence and uniqueness theory and how it is developed for ODEs, the Poisson equation, and the heat equation.
- be able to understand the finite difference and finite element approximations and prove error estimates.
- understand fundamental results about the spectral theory of linear operators.
- be familiar with some stochastic PDEs and how the above theory is extended to the stochastic case.

Future topics requiring this course unit

None.

Syllabus

1. **Introduction.** Well posedness. Review of Cauchy sequences. Banach and Hilbert spaces. [2 lectures]
2. **ODEs.** Existence of solution for the initial value problem. Lipschitz property. Contraction mapping theorem. Gronwall inequality. Convergence of Euler discretisation. [5 lectures]
3. **Model Diffusion Problem.** Sobolev spaces and generalised derivatives. Classical and weak solutions of model diffusion problem. Existence and uniqueness of solution. Riesz Representation Theorem. [6 lectures]
4. **Galerkin Approximation and the Finite Element Method.** Classical and weak solutions. Galerkin approximation and convergence. Finite element spaces. [5 lectures]
5. **Linear operators.** Bounded linear operator. Uniform boundedness theorem. Symmetric and compact operators. Integral operators. Hilbert-Schmidt theorem. Application to the Sturm Liouville problem. Unbounded operators.[6 lectures]

6. **Introducing randomness.** Probability space. Random variables. Gaussians. Examples of stochastic PDEs and numerical approaches. [2 lectures]

Textbooks

- Howard Elman, David Silvester and Andy Wathen, *Finite Elements and Fast Iterative Solvers*, Oxford University Press, Oxford 2005, ISBN 0-19-852868-X (paperback).
- Brenner, Scott, *The Mathematical Theory of Finite Element Methods - 2nd Edition*, Springer, 2002, ISBN 0-387-95451-1
- Renardy and Rogers, *An Introduction to Partial Differential Equations*, Springer 1993

Teaching and learning methods

27 lectures (two or three lectures per week) with a weekly examples class. In addition should expect to spend at least seven hours each week on private study.

Assessment

Mid-semester coursework: 25%
End of semester examination: three hours weighting 75%

Arrangements