MANCHESTER 1824

MATH20401 - 2009/2010

General Information

- Title: Partial Differential Equations and Vector Calculus
- Unit code: MATH20401
- Credit rating: 20
- Level: 2
- Pre-requisite units: MATH10121 or MATH10131, MATH10222 or MATH10232
- Co-requisite units:
- School responsible: Mathematics
- Members of staff responsible: Dr. David Harris and Dr. Tony Shardlow

Unit specification

Aims

This course introduces students to (i) analytical and numerical methods for solving partial differential equations (PDEs), and (ii) concepts and methods of vector calculus. It builds on the first year core applied mathematics courses to develop more advanced ideas in differential and integral calculus.

Brief description

The first half of the course consists of an introduction to the important topics of Fourier series, partial differential equations and analytical methods for solving first and second order PDEs (including Laplace's equation and the heat and wave equations). The second half of the course includes an introduction to the specialist topic of numerical analysis (finite difference methods for solving differential equations) together with an introduction to fundamental results underlying vector calculus (the divergence theorem and Green's and Stokes' theorems). The tools introduced in the course will be essential for understanding subsequent applied mathematics and numerical analysis options in the remaining semesters of the BSc and MMath degree programmes.

Intended learning outcomes

On completion of this unit successful students will be able to:

- convert Cartesian coordinates into cylindrical and spherical polar coordinates and solve problems expressed in these coordinate systems;
- understand functions of several variables, their partial differentiation, integration, and their geometrical interpretation;
- understand the basic concept of orthogonal functions;
- compute Fourier series, Fourier sine series and Fourier cosine series of piecewise continuous functions;
- solve first order parial differential equations using the method of characteristics;
- recognise classical PDEs describing physical processes such as diffusion, wave propagation and electrostatics;
- choose appropriate boundary and initial conditions for PDEs;

- classify second-order PDEs as elliptic, hyperbolic or parabolic;
- solve analytically, using the method of separation of variables, the heat and wave equations (in one space variable) and Laplace's equation (in two space variables) on rectangular and circular domains;
- solve numerically, via finite difference schemes, the heat equation in one space variable;
- solve convection-diffusion equations numerically using upwind finite difference schemes;
- compute elements of surface and volume in different coordinate systems;
- evaluate line, surface and volume integrals over domains using various coordinate systems;
- use grad, div and curl operator notation and relate key identities to properties of vector fields;
- understand, and use the Divergence, Green's and Stokes' theorems;
- understand the concept of a tensor and be able to use indicial notation and the summation convention.

Future topics requiring this course unit

The material in this course unit is essential for all applied mathematics options in subsequent semesters.

Syllabus

1. Cartesian, curvilinear coordinates (including cylindrical, spherical polars). Functions of several variables and partial derivatives. Surfaces, unit vectors, elements of surface/volume. Integrals of functions of several variables. Periodic functions. Orthogonality. Fourier coefficients and Fourier's theorem. Parseval's theorem. First order partial differential equations. Method of characteristics. Shocks. [10 lectures]
2. Partial Differential Equations: linearity, homogeneity, order. Semilinear, quasilinear and nonlinear PDEs. Principle of Superposition. Classification of PDEs into elliptic, parabolic, and hyperbolic equations. Second order PDEs: Laplace's, diffusion and wave equations. Separation of variables. Solution of second order ordinary differential equations (e.g. Bessel's equation) by series expansion. Initial value problems for the heat and wave equations. Eigenfunction series and normal modes. Laplace's equation separated in Cartesian and plane polar coordinates. [10]
3. Vector Calculus. Line, surface and volume integrals. Scalar and vector fields: differential and integral calculus. Field lines and field surfaces, the general field surface of a vector field. Grad, div and curl in Cartesian and curvilinear coordinates. Identities. Divergence, Green's and Stokes' theorems. Indicial notation, summation convention. Zero, first, second and higher order tensors. [11]
4. Numerical Analysis of differential equations. Finite diference methods. Heat equation in one space variable. Explicit and implicit schemes, truncation error, stability and convergence. Crank-Nicholson scheme. Convection-diffusion problems, upwind differencing. [11]

Textbooks

- Endre Suli and David Mayers, *An Introduction to Numerical Analysis*, Cambridge University Press 2003.
- W. Strauss, *Partial differential equations: an introduction*, John Wiley (2008).
- M. Renardy and R. C. Rogers, *An introduction to partial differential equations*, Springer (2004).
  Others to be notified later.

Learning and teaching processes

Four lectures and one or two examples classes each week. In addition students should expect to do at least four hours private study each week for this course unit.

Assessment

Two tests both worth 10%: the first in week 7 (on material covered in section 1 of syllabus) and the other in week 8 (on material in section 2 of the syllabus)
3 hours end of semester examination; Weighting within unit 80%

Arrangements