MATH35051 - 2009/2010

General Information

- Title: Singularities, Bifurcations and Catastrophes
- Unit code: MATH35051
- Credit rating: 10
- Level: 3
- Pre-requisite units: MATH20201 Algebraic Structures I, MATH20132 Calculus of several variables.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. J. Montaldi

Unit specification

Aims

This lecture course aims to introduce students to the fundamental ideas of classifying and understanding singularities of functions, and to enable them to apply these notions to bifurcation problems.

Brief description

Singularity theory is the study of the local structure of maps $\boldsymbol{R}^n$ to $\boldsymbol{R}^p$ (or $\boldsymbol{C}^n \to \boldsymbol{C}^p$), particularly when the conditions of the inverse or implicit function theorems fail, and how this structure changes as the map is deformed. In this course we will concentrate on the scalar case (p=1), with the level 4 version including cases with p>1. Such questions arise in many applications, particularly in bifurcation theory and what is sometimes called catastrophe theory.

Intended learning outcomes

On successful completion of this course unit students will be able to:

- use algebraic techniques to compute the codimension of a singularity or a bifurcation;
- classify degenerate critical points;
- find a versal unfolding of a singularity;
- apply the theory of unfoldings to study bifurcations in physical systems.

Future topics requiring this course unit

Syllabus

1. Introduction; geometry of implicit function theorem and linearly adapted coordinates (submanifolds), critical points and the Morse Lemma, Splitting Lemma. [4 lectures]
2. Germs of smooth functions $(\boldsymbol{R}^n,0) \to \boldsymbol{R}$, polynomial ideals and Newton diagram, Nakayama Lemma. [3]

3. Jacobian ideal, codimension of function germs, Right-equivalence, Finite determinacy. [4]
4. Classification and recognition principles. [3]
5. Unfoldings, geometry of “elementary catastrophes”, Catastrophe set and its projection. [3]
6. Applications and further topics: Elastic buckling, ship stability, optics, ”imperfect” bifurcations, symmetry. [5]

Textbooks

- T. Poston and I.N. Stewart, Catastrophe Theory and its Applications. Dover.
- Th. Bröcker, Differentiable Germs and Catastrophes, LMS Lecture Notes, CUP.

Learning and teaching processes

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Assessment

Mid-semester test: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements