MATH31072/MATH41072 - 2010/2011

General Information

- Title: Algebraic Topology
- Unit code: MATH31072/MATH41072
- Credits: 10 (MATH31072), 15 (MATH41072)
- Prerequisites: MATH20212 *Algebraic Structures 2*, MATH31051 *Introduction to Topology.*
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Jelena Grbic

Specification

Aims

This lecture course has for its aim the further development of the concepts introduced in the course Introduction to Topology, emphasizing topics coming from Algebraic Topology. The course will introduce students to the basic concepts of homotopy and homology theory, and explain the need for different algebraic invariants of topological spaces.

Brief Description of the unit

This course illustrates how algebra and topology interact in the field of Algebraic Topology, by considering algebraic invariants of topological spaces, that is, algebraic objects that are preserved by homeomorphisms. In Algebraic Topology, one tries to relate algebraic invariants to topological spaces and continuous maps in order to say something about topological/geometrical problems. The invariants we describe are usually groups. For example, they might be used in certain cases to show that two topological spaces are not homeomorphic.

The first half of the course unit introduces the basic definitions and standard examples of homotopy theory. We study one of the simplest and most important algebraic invariants of Algebraic Topology, the fundamental group, which creates an algebraic image of the space of maps from the circle to the topological space. Special accent is laid upon different methods for calculating fundamental groups.

The second half further explores homology theory, which is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications.

Learning Outcomes

On successful completion of this course unit students will be able to:

- understand the concept of homotopy and know its basic properties;
- calculate the fundamental group of a simple topological space;
- recognize whether or not two topological spaces are homotopic;
- understand different approaches in defining homology groups;
- calculate the homology groups of naturally occurring topological spaces;
- use homology groups to say something about the homotopy type of a topological space;
- understand the structural differences between homology and homotopy.

Future topics requiring this course unit

Syllabus

1. Homotopy,
2. The Fundamental Group;
3. Applications of the Fundamental Group;

4. The Van Kampen Theorem;
5. Simplicial Homology;
6. Singular Homology;
7. Relative Homology Groups;
8. Computation of Homology;
9. The Meyer-Vietoris Sequence.

For MATH41072 the lectures will be enhanced by additional reading on related topics.

Textbooks

The following book contains most of the material in the course and much more.

- A. Hatcher, *Algebraic Topology*. (free download)

Additional Reading

- G. Bredon, *Topology and Geometry.*
- J. Munkres, *Topology* (2nd Edition).

Teaching and learning methods

Three classes each week which will include opportunities to discuss problems from the Problems Sheets. In addition students should expect to do at least four hours private study each week for this course unit (seven hours for MATH41072).

Assessment

Mid-semester coursework: weighting 16% (MATH31072), 10% (MATH41072),
End of semester examination: two hours weighting 84% (MATH31072), three hours weighting 90% (MATH41072)

Arrangements