MANCHESTER 1824

MATH44011 - 2010/2011

General Information

- Title: Asymptotic Expansions and Perturbation Methods
- Unit code: MATH44011
- Credits: 15
- Prerequisites: None
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. J. Gajjar

Specification

Aims

To demonstrate the power and the beauty of modern asymptotic methods via discussing the main ideas and approaches used in the theory of asymptotic expansions to simplify and to solve different mathematical problems which involve large or small parameters.

Brief Description of the unit

The development of the theory of asymptotic expansions, which serves as a foundation for perturbation methods, is one of the most important achievements in applied mathematics in the twentieth century. Perturbation methods represent a very powerful tool in modern mathematical physics and in particular, in fluid dynamics. This course unit will introduce students to a range of modern asymptotic techniques and illustrate their use in model problems involving ordinary and partial differential equations.

Learning Outcomes

On successful completion of the course unit the students will acquire thorough understanding of fundamental ideas used in the theory of asymptotic expansions and will develop appropriate practical skill in applying asymptotic methods for analysing mathematical and physical problems with small or large parameters.

Future topics requiring this course unit

None.

Syllabus

1. Asymptotic expansions. Taylor expansion as a conventional converging power series and as an example of an asymptotic expansion . Asymptotic expansions for definite integrals with the upper or lower limits of integration depending on small or large parameters. Functions defined by real integrals. Laplace's method for definite integrals the integrand being of the form $f(t)\exp(\lambda t)$, where the parameter λ is large; Watson's Lemma. Generalisation for functions defined by contour integrals. Steepest descent. Applications.
2. Regular asymptotic expansions for functions depending on the coordinate x, scalar or vector, and on a small parameter ε. Solution of ordinary and partial differential equations with small parameters.
3. Singular perturbations (algebraic example). Notion of the boundary layer. Inner and outer solutions. Overlap region. Matching of the asymptotic expansions. Ordinary differential equations with singular perturbations. Methods to determine location of the boundary layer.
4. Method of multiple scales. Quasi-periodic solutions of second order ordinary differential equations developing non-uniformity at large time. Uniformly valid solutions. Amplitude equations. WKB Method.
5. Examples from Fluid Dynamics. [Number of applications covered will depend on time available.] Low Reynolds number flows. Ackeret formula for inviscid supersonic flows. Thin aerofoil theory for subsonic flows. Large Reynolds number solutions of the Navier-Stokes equations. Blasius boundary layer on a flat plate surface. Non-uniformities of the Blasius solution near the flat plate leading edge. Second order approximation for the solution near the trailing edge. Triple-deck theory for the trailing edge flow. Self-induced separation of the boundary layer in supersonic flow. Principle of dominant balance, 'inspector's analysis'. Linear solutions for the triple-deck equations. Supersonic and subsonic corner flows.

Textbooks

- R. Wong, Asymptotic Approximation of Integrals, Academic Press 1989.
- E.J. Hinch, Perturbation Methods, Cambridge 1991.
- O.M. Bender and S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers.
- M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press 1964.
- J. Kevorkian and J.D. Cole, Perturbation Methods in Applied Mathematics, Springer 1985.
- A.H. Nayfeh, Perturbation Methods, Wiley 1973.

Teaching and learning methods

Three hours each week with 28-30 lectures and 6-8 examples classes in total. In addition students should expect to do at least seven hours private study each week for this course unit.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: three hours weighting 80%

Arrangements