MATH34042 - 2010/2011

General Information

- Title: Discrete Time Dynamical Systems
- Unit code: MATH34042
- Credits: 10
- Prerequisites: None
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. P. Glendinning

Specification

Aims

To introduce nonlinear discrete time dynamical systems and study some of their properties, in particular the kinds of dynamics they can exhibit.

Brief Description of the unit

This course introduces discrete time dynamical systems (iterated mappings) and analyses them using the sort of qualitative approaches developed for continuous time systems in MATH10202 or MATH10232. Mappings of the interval [0, 1] to itself are studied in detail; these are simple examples of discrete time systems but they can show remarkably complex dynamical behaviour, including chaotic dynamics. The existence of fixed points and periodic points is explored, and the way these change as the system changes (bifurcation theory) is investigated. The basic ideas of symbolic dynamics as a way of analysing dynamical systems is introduced, and the method is used to show some simple maps have chaotic behaviour.

Learning Outcomes

On successful completion of this course unit students will

- have acquired a basic understanding of discrete time dynamical systems on the interval;
- be able to find the fixed and periodic points of simple dynamical systems on the interval, and determine their stability;
- have some familiarity with some of the simpler bifurcations that fixed and periodic points can undergo;
- have some familiarity with the notion of self-similar fractals, and how they arise as attractors.

Future topics requiring this course unit

None.

Syllabus

1. Introduction to discrete time systems. Simple examples and applications. Fixed and periodic points. Stability. [3 lectures]
2. One dimensional systems. Graphical analysis. Stability of fixed and periodic points. Basin of attraction. The logistic map. The period doubling cascade. Sensitive dependence on initial conditions. [6]
3. Itineraries and Sarkovskii's theorem. Chaos. Transition graphs and symbolic dynamics. [4]
4. Fractals. Cantor set, iterated function system, dynamics and fractals, fractional dimension. [5]
5. Bifurcation of fixed points. Bifurcation diagrams. [4]

Textbooks

- K. T. Alligood, T. D. Sauer and J. A. Yorke, *Chaos - an Introduction to Dynamical Systems*, Springer-Verlag 1996.

Teaching and learning methods

Two lectures and one examples class each week. In addition students should expect to spend at least four hours each week on private study for this course unit.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements