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MATH45122 - 2010/2011

General Information

- Title: Non-Linear Waves
- Unit code: MATH45122
- Credits: 15
- **This course unit cannot be taken in the same semester as MATH35012 *Wave Motion*.**
- Prerequisites: None
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Prof. N. Gray

Specification

Aims

The aim of the course is to introduce some ideas associated with nonlinear wave propagation such as wave steepening, shock formation, dispersion, and soliton properties, arising in simple physical systems.

Brief Description of the unit

Waves occur in many physical systems, and in the absence of nonlinear effects the appropriate wave equation can usually be solved analytically. Nonlinear effects bring new physics into play and shocks (jumps in solutions) can occur. The ideas used will be applied to explain, for example, bores on rivers and traffic flow problems. The breakdown of nonlinear wave systems will also be described. The methods used apply to a large variety of situations of practical importance.

Learning Outcomes

On successful completion of the course unit students will be able to

- understand the differences in the solution properties and physics of physical systems governed by the kinematic wave equation, the K - dV equation, Burger's equation, and the shallow water equations;
- solve $u_t + c(u)u_x = 0$ for given initial data and be able to identify the formation of shocks;
- understand how breaking waves in two-dimensions can be represented in terms of shock waves;
- solve the shallow water equations using the method of characteristics for simple flows;
- perform a phase plane analysis for the K - dV and related equations to identify travelling wave solutions, solitary wave solutions.

Future topics requiring this course unit

None.

Syllabus

1. The hyperbolic wave $u_{tt} = c_0^2\Delta^2 u$, $u_t \pm c_0 u_x = 0$; wave forms; Fourier synthesis; dispersion; $C(k) = dw/dk$, group velocity; diffusion, e.g. Burger's linear equation $u_t + c_0 u_x = \nu u_{xx}$.
2. First order wave equation $u_t + c(u)u_x = 0$; characteristics; conservation ideas; conservation forms; traffic flow models. Waves in other physical systems.
3. First order equations in two-dimensions; breaking waves and their representation in terms of shocks.
4. Shallow water wave theory; the nonlinear equations; wave breaking, dam break problems, via characteristics; normal and oblique shocks, linearisation and check against linear theory, and linear irrotational theory.
5. Irrotational water wave theory to obtain the Boussinesq equations; steady solutions of the Boussinesq equations; derivation of the Korteweg-de Vries equation from Boussinesq equations; conservation laws for K - dV; analytical solution of K - dV equation; the soliton.

Textbooks

- P.G. Drazin and R.S. Johnson, *Solitons, An Introduction*, CUP 1989.
- G.B. Whitham, *Linear and Non-linear Waves*, Wiley 1974.
- J. Stoker, *Water Waves*, Wiley Interscience 1957.
- L. Debnath, *Nonlinear Water Waves*, Academic Press 1994.

Teaching and learning methods

22 lectures and 11 examples classes. In addition students should expect to do at least seven hours private study each week for this course unit.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: two and a half hours weighting 80%

Arrangements