MATH45051 - 2010/2011

General Information

- Title: Singularities, Bifurcations and Catastrophes
- Unit code: MATH45051
- Credit rating: 15
- Level: 3
- Pre-requisite units: MATH20201 Algebraic Structures I, MATH20132 Calculus of several variables.
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. J. Montaldi

Unit specification

Aims

This lecture course aims to introduce students to the fundamental ideas of classifying and understanding singularities of functions, and to enable them to apply these notions to bifurcation problems.

Brief description

Singularity theory is the study of the local structure of maps $\boldsymbol{R}^n$ to $\boldsymbol{R}^p$ (or $\boldsymbol{C}^n \to \boldsymbol{C}^p$), particularly when the conditions of the inverse or implicit function theorems fail, and how this structure changes as the map is deformed. In this course we will concentrate on the scalar case (p=1), with the level 4 version including cases with p>1. Such questions arise in many applications, particularly in bifurcation theory and what is sometimes called catastrophe theory.

Intended learning outcomes

On successful completion of this course unit students will be able to:

- use algebraic techniques to compute the codimension of a singularity or a bifurcation;
- classify degenerate critical points;
- find a versal unfolding of a singularity;
- apply the theory of unfoldings to study bifurcations in physical systems.

Future topics requiring this course unit

Syllabus

1. **Families of functions:** Critical points, Hessian and (non)degeneracy, Morse lemma, catastrophe set, discriminant, *Singular points of maps, families of maps.* [3]
2. **Changes of coordinates and submanifolds:** diffeomorphisms, inverse function theorem, germs, implicit function theorem, submanifolds, parametrization. *Linearly adapted coordinates, transversality, Lyapunov-Schmidt reduction.* [3]

3. **Some algebra:** ring of germs of smooth functions, maximal ideal, Nakayama's lemma, Newton diagram, finite codimension ideals. *Filtrations, modules over rings of polynomials.* [3]
4. **Critical points:** Jacobian ideals, codimension, right equivalence, finite determinacy, splitting lemma, classification of critical points. *Other singularity-theoretic equivalences, their tangent spaces, codimension and finite determinacy.* [7]
5. **Unfoldings:** Families of functions as unfoldings, versal unfoldings, versality theorem for right-equivalence. *Versality for other equivalences.* [4]
6. **Applications:** Illustration of types of application, such as the geometry of curves and surfaces, gravitational catastrophe machine, ship stability, or other examples. *Bifurcation of equilibria, pitchfork bifurcation, Hopf bifurcation* [2]

Items in italics are particular to the level four version, some of which will be covered in supplementary lectures, and some by extra reading.

Textbooks

- V. Arnold, V. Goryunov, O.V. Lyashko & V.A. Vasil'ev, Singularity Theory I. Springer. 1993.
- C.G. Gibson, Singular Points of Smooth Mappings. Pitman, 1979.
- T. Poston and I.N. Stewart, Catastrophe Theory and its Applications. Dover.
- Th. Bröcker, Differentiable Germs and Catastrophes, LMS Lecture Notes, CUP.

Learning and teaching processes

Two lectures and one examples class each week, supplemented with extra reading provided by the lecturer plus 4 or 5 extra lectures. In addition students should expect to spend at least seven hours each week on private study for this course unit.

Assessment

Mid-semester test: weighting 10%
End of semester examination: three hours, weighting 90%

Arrangements