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MATH20722 - 2011/2012

General Information

- Title: Foundations of Modern Probability
- Unit code: MATH20722
- Credit rating: 10
- Level: 2
- Pre-requisite units: MATH10141, MATH20701
- Co-requisite units: None
- School responsible: Mathematics
- Member of staff responsible: Dr. J. Bagley

Unit specification

Aims

The programme unit aims to

- provide the basic knowledge of facts and methods needed to state and prove the law of large numbers and the central limit theorem;
- introduce fundamental concepts and tools needed for the rigorous understanding of third and fourth level course units on probability and stochastic processes including their applications (e.g. Financial Mathematics).

Brief description

The law of large numbers and the central limit theorem are formulated and proved. These two results embody the most important results of classical probability theory having an endless number of applications. A good understanding of MATH10101 or 10111, 20101 or 20111, and 20701 is required.

Intended learning outcomes

On completion of this unit successful students will:

- understand the meaning and proof of the law of large numbers;
- understand the meaning and proof of the central limit theorem;
- be able to apply the methods of proof to related problems.

Future units for which this unit is desirable but not essential.

MATH37001 Martingales with Applications to Finance (level 3 semester 1)
MATH47101 Stochastic Calculus (level 4 semester 1)
MATH47112 Brownian Motion (level 4 semester 2)

Syllabus

1. Probability measures. Probability spaces. Random variables. Random vectors. Distribution functions. Density functions. Laws. The two Borel-Cantelli lemmas. The Kolmogorov 0-1 law. [4 lectures]
2. Inequalities (Markov, Jensen, Hölder, Minkowski). Modes of convergence (almost sure, in probability, in distribution, in mean). Convergence relationships. [4]
3. Expectation of a random variable. Expectation and independence. The Cesàro lemma. The Kronecker lemma. The law of large numbers (weak and strong). [5]
4. Fourier transforms (characteristic functions). Laplace transforms. Uniqueness theorems for Fourier and Laplace transforms. Convergence of characteristic functions: the continuity theorem. The central limit theorem. [6]
5. Brownian motion as the weak limit of the random walk. Donsker's Theorem. [3]

Textbooks

- D. Williams, *Probability with Martingales*, Cambridge University Press 1991.
- A.N. Shiryaev, *Probability*, Springer-Verlag 1996.
- G.R. Grimmett and D.R. Stirzaker, *Probability and Random Processes*, Oxford University Press 1992.

Learning and teaching processes

Two lectures and one examples class each week. In addition students are expected to do at least four hours private study each week on this course unit.

Assessment

Mid-semester coursework: weighting 20%
End of semester examination: two hours weighting 80%

Arrangements