MATH42001/32001 - 2011/2012

General Information

- Title: Group Theory
- Unit code: MATH32001/42001
- Credits: 10 (MATH32001), 15 (MATH42001)
- Prerequisites: MATH20212 *Algebraic Structures 2*
- Co-requisite units: None
- School responsible: Mathematics
- Member of staff responsible: Prof. Peter Rowley

Specification

Aims

This lecture course aims to introduce students to some more sophisticated concepts and results of group theory as an essential part of general mathematical culture and as a basis for further study of more advanced mathematics.

Brief Description of the unit

The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outstanding beauty. It takes just three simple axioms to define a group, and it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts and results of Group Theory.

Learning Outcomes

On successful completion of this course unit students will have acquired

- a sound understanding of the classification of finitely generated abelian groups,
- knowledge of some fundamental results and techniques from the theory of finite groups,
- knowledge of group actions on sets, simple groups, Sylow's theorems and various applications of Sylow's theorems.

Future topics requiring this course unit

MATH42122 *Galois Theory.*

Syllabus

1. Revision of basic notions (subgroups and factor groups, homomorphisms and isomorphisms), generating sets, commutator subgroups. [2 lectures]
2. Abelian groups, the Fundamental Theorem on finitely generated abelian groups. [4]
3. The Isomorphism Theorems. [3]
4. Simple groups, the simplicity of the alternating groups. [3]
5. Composition series, the Jordan-Hölder Theorem. [2]
6. Group actions on sets, orbits, stabilizers, the number of elements in an orbit, Burnside's formula for the number of orbits, conjugation actions, centralizers and normalizers. [5]
7. Sylow's Theorems, groups of order pq, pqr. [3]

For MATH42001 the lectures will be enhanced by additional reading on related topics:

- commutators and some of their elementary properties;
- proof of Ito's theorem which states that a group which is the product of two abelian groups has trivial second derived subgroup;
- finite abelian groups, including a proof that the rank and torsion coefficients determine the abelian group uniquely;

- proving that A_n for n greater than or equal to 5 is a simple group;
- proving that a simple group of order 60 must be isomorphic to A_5.

Textbooks

Recommended text:

- John B Fraleigh, A First Course in Abstract Algebra, (5th edition), 1967, Addison-Wesley.

Teaching and learning methods

Two lectures and one examples class each week.

Four hours private study each week (seven hours for MATH42001).

Assessment

Coursework: weighting 10% (MATH32001), 7% (MATH42001)
End of semester examination: MATH32001, two hours, weighting 90%; MATH42001 three hours, weighting 93%.

Arrangements