MANCHESTER 1824

MATH10202 -2011/2012

General Information

- Title: Linear Algebra A
- Unit code: MATH10202
- Credits: 20
- Prerequisites: A-Level Mathematics or equivalent, MATH10101 (for proof by contradiction, proof by induction, sets and functions, and finite fields), MATH10121 (for complex numbers and vector algebra)
- Co-requisite units: None
- School responsible: Mathematics
- Members of staff responsible: Dr. Carolyn Dean

Specification

Aims

The aims of this course are to introduce the basic ideas and techniques of linear algebra for use in many other lecture courses. The course will also introduce some basic ideas of abstract algebra and techniques of proof which will be useful for future courses in pure mathematics.

Brief Description of the unit

This major core course aims at introducing students to the fundamental concepts of linear algebra culminating in abstract vector spaces and linear transformations. The first part covers systems of linear equations, matrices, and some basic concepts of the theory of vector spaces in the concrete setting of real linear *n*-space, R^n. The second part briefly explores orthogonality, and then goes on to a full discussion of abstract vector spaces over arbitrary fields and of linear transformations. The subject material is of vital importance in all fields of mathematics and in science in general.

Learning Outcomes

On successful completion of this module students will

- be able to solve systems of linear equations by using Gaussian elimination to reduce the augmented matrix to row echelon form or to reduced row echelon form;
- understand the basic ideas of vector algebra: linear dependence and independence and spanning;
- be able to apply the basic techniques of matrix algebra, including finding the inverse of an invertible matrix using Gauss-Jordan elimination;
- be able to find and apply the LU factorization of a matrix;
- know how to find the row space, column space and null space of a matrix, to find bases for these subspaces and be familiar with the concepts of dimension of a subspace and the rank and nullity of a matrix, and to understand the relationship of these concepts to associated systems of linear equations;
- be able to find the eigenvalues and eigenvectors of a square matrix using the characteristic polynomial and will know how to diagonalize a matrix when this is possible;
- be able to use the Gram-Schmidt process to find an orthogonal basis for a subspace of real linear *n*-space;
- be able to find the orthogonal complement of a subspace;
- be able to recognize and invert orthogonal matrices;
- be able to orthogonally diagonalize symmetric matrices;
- be able to diagonalize quadratic forms;
- be familiar with the general notions of a vector space over a field and of a subspace, linear independence, dependence, spanning sets, basis and dimension of a general subspace;
- be able to find the change-of-basis matrix with respect to two bases of a vector space;
- be familiar with the notion of a linear transformation, its matrix with respect to bases of the domain and the codomain, its range and kernel, and its rank and nullity and the relationship between them;

- be familiar with the notion of a linear operator and be able to find the eigenvalues and eigenvectors of an operator.

Future topics requiring this course unit

Most mathematics course units in pure mathematics, applied mathematics and statistics.

Syllabus

1. **Linear equations:** systems of linear equations (Poole 2.1), matrices and row echelon form (Poole 2.2), Gaussian elimination (Poole 2.2), Gauss-Jordan elimination (Poole 2.2). [4 lectures]
2. **Vectors and matrices:** linear combinations of vectors (Poole 2.3), linear independence of vectors (Poole 2.3), matrix operations (Poole 3.1), matrix algebra (Poole 3.2). [4]
3. **Elementary matrices:** the inverse of a matrix (Poole 3.3), finding the inverse of a matrix by Gauss-Jordan elimination (Poole 3.3), LU factorization (Poole 3.4). [3]
4. **Subspaces and linear transformations:** subspaces (Poole 3.5), bases and dimension (Poole 3.5), rank and nullity (Poole 3.5), coordinates and linear transformations (Poole 3.6), applications of linear transformations (Poole 3.6). [5]
5. **Diagonalization of matrices:** eigenvalues and eigenvectors (Poole 4.1), determinants (Poole 4.2), the characteristic equation (Poole 4.3), diagonalization of matrices (Poole 4.4), applications of diagonalizability (Poole 4.6). [5]
6. **Orthogonality:** orthogonal and orthonormal sets and bases (Poole 5.1), Gram-Schmidt orthogonalization process (Poole 5.3), orthogonal matrices (Poole 5.1), orthogonal complements, Orthogonal Decomposition Theorem, orthogonal projections (Poole 5.2), applications to matrices, fundamental subspaces, Rank Theorem (Poole 5.2), orthogonal diagonalization of symmetric matrices, Spectral Theorem (Poole 5.4), quadratic forms, Principal Axis Theorem, positive definite forms and matrices (Poole 5.5). [7]
7. **Vector spaces:** vector spaces and subspaces, definition of vector spaces, examples, subspaces, subspace criterion, *sum of subspaces, spanning sets (Poole 6.1), linear independence, basis, dimension, in a finite-dimensional vector space: every spanning set contains a basis, every linearly independent set can be extended to a basis, any two bases have the same number of elements [Basis Theorem], subspaces are finite dimensional, *dimension of the sum of two subspaces (Poole 6.2), coordinates (Poole 6.2), change of bases, change-of-basis matrices, Gauss-Jordan method for computing change-of-basis matrices (Poole 6.3), linear transformations, definition and examples, composition, inverse (Poole 6.4), kernel and range of a linear transformation, kernel, range, rank and nullity, Rank Theorem, one-to-one and onto linear transformations (Poole 6.5), universal property of vector spaces, isomorphisms of vector spaces (Poole 6.5), matrices of linear transformations, definition and elementary properties, matrices of composites and inverses (Poole 6.6), change-of-basis and similarity (Poole 6.6), diagonalization of linear transformations (Poole 6.6). [15]

* indicates topics that are not explicitly contained in Poole.

Textbooks

The course is based on the course text which students are expected to buy:

- David Poole, *Linear Algebra: a Modern Introduction,* Thomson, second edition 2006, International Student Edition: ISBN 0-534-40596-7. (There is a useful websiteassociated with the text).

Teaching and learning methods

Four lectures and one supervision class each week.

Assessment

Attendance at supervisions: weighting 5%
Submission of coursework at supervisions: weighting 5%
In-class test: weighting 15%
Three hours end of semester examination: weighting 75%

Arrangements